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TECHNICAL REPORT AFATL-TR-74-156



# PROJECTILE MEASUREMENTS AND INSTRUMENTATION LABORATORY MASS PROPERTY MEASUREMENTS

INTERIOR/EXTERIOR BALLISTICS BRANCH
GUNS, ROCKETS AND EXPLOSIVES DIVISION



SEPTEMBER 1974

FINAL REPORT: September 1974



AIR FORCE ARMAMENT LABORATORY

AIR FORCE SYSTEMS COMMAND • UNITED STATES AIR FORCE

EGLIN AIR FORCE BASE, FLORIDA

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The basic theory and techniques for measuring the mass properties of test models as performed in the Projectile Measurements and Instrumentation Laboratory at Eglin Air Force Base are described. The intent is to provide an insight into the principles on which mass property measuring instruments are designed and an understanding of the pitfalls to be avoided and the difficulties involved in obtaining a correct measurment of physical quantities, with the types of instruments available.

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Item 19 (Continued):

Top Loading Balance
Center of Gravity Balance
Moment of Inertia Instrument
Measurement Procedure
Dimension
Weight (Mass)
Center of Gravity
Moment of Inertia

## PREFACE

The Projectile Measurements and Instrumentation Laboratory is one of the support facilities within the Interior/Exterior Ballistics Branch (DLDL), Guns, Rockets and Explosives Division, Air Force Armament Laboratory, Eglin Air Force Base, Florida. This facility occupies a portion of Building 408, Test Area A22.

This technical report discusses the equipment and procedures used for mass property measurements in the Projectile Measurements and Instrumentation Laboratory. The equipment in this Laboratory is typical of the commercially available items, and any reference to model or manufacturer does not constitute endorsement of these products by the Air Force.

This technical report has been reviewed and is approved.

A. D. Brown Jr

ALFRED D. BROWN, JR, Colonel, USAF
Chief, Guns, Rockets and Explosives Division



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## SECTION I

## INTRODUCTION

The Projectile Measurements and Instrumentation Laboratory provides munition mass property data in support of free flight and wind tunnel tests conducted at the Ballistic/Aerodynamic Research System and the Aeroballistic Research Facility. An additional purpose is to provide research, development, and maintenance support for new instrumentation systems required for testing in these facilities. Another function of this Laboratory is to provide support as required for other groups within the Air Force Armament Laboratory (AFATL).

This report discusses the equipment available in the Projectile Measurements and Instrumentation Laboratory and the techniques used in mass property measurements. The intent is to provide an insight into the principles involved in obtaining an accurate measurement of physical quantities with the types of instruments available. The procedures used and the examples given have been determined to satisfy these systems.

This report is essentially designed to aid technicians and practicing engineers in development and research work. In some respects, the theory as presented is oversimplified for the practicing scientist and engineer engaged in associated research and development programs. However, this report is intended to be used as a guide and ready reference for personnel responsible for making mass property measurements, and some detailed development is required in order to introduce basic terms and definitions.

Section II provides an elementary discussion of the basic concepts defining mass property measurement. It also furnishes full details of standard operational methods for determining and interpreting measurement data. Step by step instructions are given where practical for attaining specified goals. Section III provides specifications for the specialized equipment used in the measurement laboratory. Emphasized in this section is a rational approach to appropriate selection of measurement instruments. Section IV is devoted to a consideration of improvements to the measuring process and measuring instruments.

Much of the material used in this report was obtained from the documents listed in the Bibliography. These publications can also be used as a source of additional information on all aspects of the measurement procedures discussed.

## SECTION II

## MASS PROPERTY MEASUREMENTS

The term ballistics has come to mean not only the specialized knowledge of propulsion devices but also the science of the motion of projectiles. Both the explosion of a charge and the resulting motion of the projectile in a gun barrel are considered in the science of interior ballistics. Exterior ballistics is a branch of applied mechanics which deals with the motion of projectiles outside the launching device and the behavior of the projectile during flight. A wide variety of techniques for studying models fired in free flight has been accumulated in ballistic studies over the years. Various methods have been developed for recording the path of a projectile over a period of time at points fixed in laboratory space. This information, along with other soon to be defined physical properties of a projectile, is applied to equations of motion. Since the projectile can be a scaled model of some larger configuration, it will be referred to as a test object or simply object.

The equations for determining the motion of a rigid object in space are:

$$d\vec{P}/dt = \vec{F} \qquad (1)$$

$$d\vec{L}/dt = \vec{N} \qquad (2)$$

Where:

$$\vec{P} = M\vec{V} \qquad (3)$$

$$\vec{L} = \vec{I}\cdot\vec{\omega} \qquad (4)$$

$\vec{F}$ and $\vec{N}$ are the total force on the object and the total torque about a suitable point of reference, i.e., center of gravity. $\vec{I}$ and $\vec{\omega}$ are the inertia tensor and the angular velocity about the center of gravity point. This applies to an unconstrained object moving in space.

Equations (2) and (4) for the rotation of a rigid object bear a formal analogy to Equations (1) and (3) for the translational motion of a mass M. In the more general case, $\vec{F}$ and $\vec{N}$ each depend on both position and orientation and must be solved simultaneously as six coupled equations in some suitable set of coordinates. No attempt is made here to treat or present this case. However, it becomes apparent that several terms in the equations can be measured and applied in the form of constants; these are properties associated with the mass of an object, i.e.,

(1) it occupies space (dimensional measurement), (2) the vector force of gravity gives a characteristic force (weight), (3) the point through which the resultant weight is measured can be located (center of gravity), and (4) the tensor quantity about the center of gravity of the object can be measured (moment of inertia). These properties will be grouped into a basic term referred to as a mass property. A more general definition is given for the procedures used in measuring the mass properties.

Mass property measurement concerns obtaining physical parameters of an object for analysis. The measurement of physical quantities provides the sole source of qualitative information about processes in the universe of physical systems. The measurements are generally in the form of numbers corresponding to position or in the form of counts of successive events. These values will rarely be the numerical values of the physical quantities directly. Those values must be deduced by correction and combination of readings on which calculations may be based.

## 1. DIMENSIONAL MEASUREMENTS

Instruments are available in innumerable variety for dimensional measurement. The methods discussed are capable of a reasonable degree of precision for dimension measurements ranging from about one millionth to one hundred inches.

Basically, there are three major approaches to dimensional measurement: mechanical, optical, and electrical. In most cases, some combination of the three major approaches is used. As in other kinds of measurements, there is a growing tendency to use the assistance of electrical methods for indicating, transmitting, and recording the data, and for increasing resolution and accuracy. The following discussions will attempt to classify the methods and to point out some of the more recent developments that are useful in an experimental laboratory.

In the order of increasing accuracy and resolution, the common mechanical tools for dimensional measurement are the linear scale, the scale with a vernier, the micrometer screw, and the multiplying lever (or gear) system as represented in the dial-gage micrometer. The accuracy of the final measurement when using these mechanical methods is dependent not only on the device but on the skill of the observer as well. The best estimate of the true value may be obtained by repeated readings and statistical treatment of the data. For the micrometer calipers and the dial gage, resolution and accuracy will be affected by the method of use, e.g., the force applied to the micrometer screw. In either of these cases, uniform spring-applied pressure can be used.

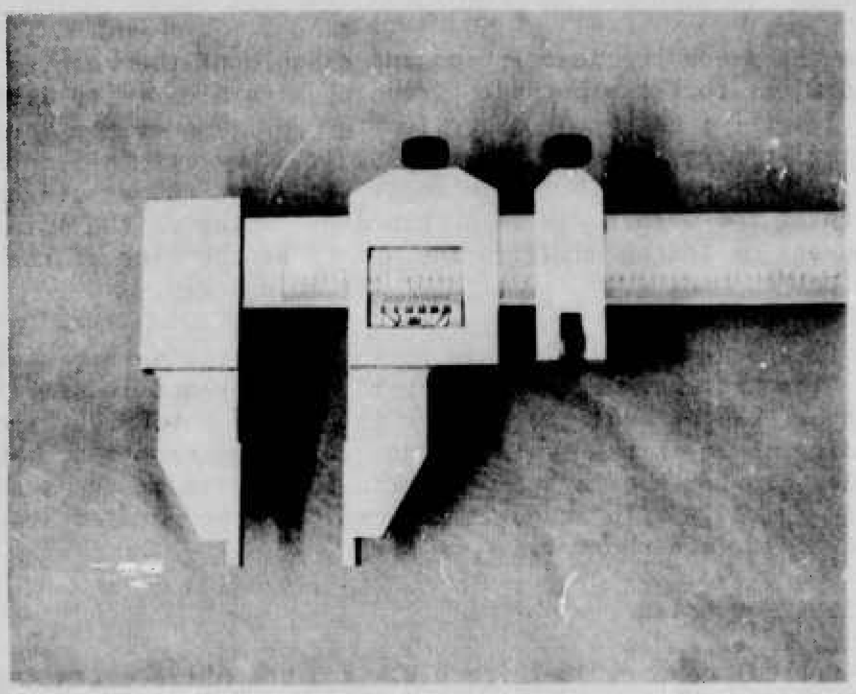
Optical comparators are used for the accurate measurement of small dimensions. All measurements are read as actual dimensions, without computation, from digital displays to within 0.001 mm.

a. Vernier Calipers

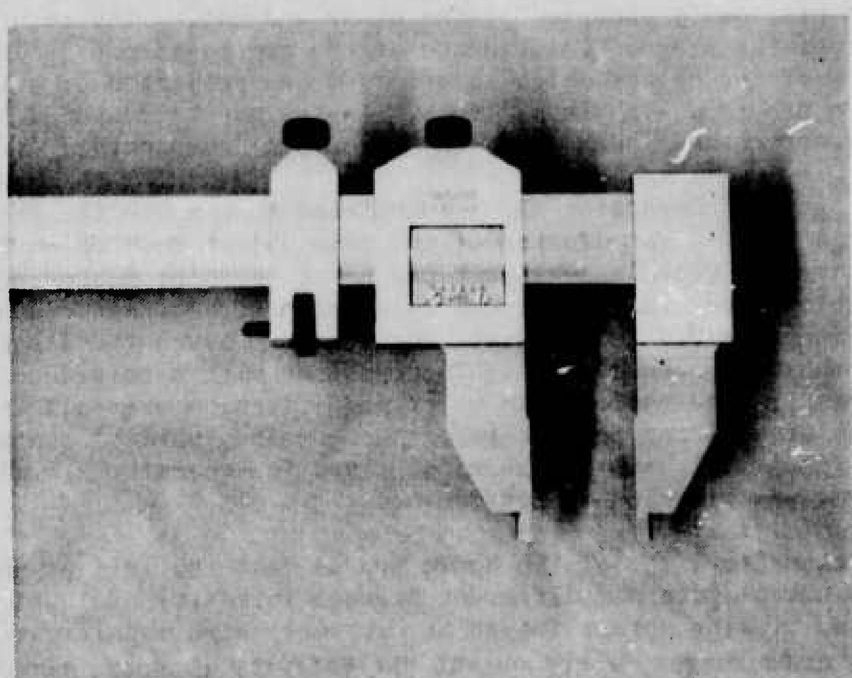
The caliper (Figure 1) consists of a main scale upon which slides a movable scale. Jaws for inside and outside measurements are affixed to the main body of the scale and to the sliding scale. One pair of jaws is used to measure the inside dimensions of objects, while the other pair is used for outside measurements. An extension to the movable frame provides a rod for depth measurements.

Fundamentally, the principle of the vernier consists of a small movable scale having a certain number of graduations which equals in combined length a different number of graduations on the stationary scale. In the vernier caliper shown in Figure 1, measurements are made in the English system on one scale, while measurements are made in the metric system on the reverse scale. On the English scale, a length of eight graduations on the movable scale equals the length of seven graduations on the stationary scale; this makes possible readings accurate to 1/128 inch. On the metric scale, a length of ten divisions of the movable scale equals the length of nine divisions on the stationary scale, making possible readings accurate to 0.1 mm.

The principle of the vernier caliper is best explained by a general example. When the jaws of the caliper are completely closed, it is observed that the zero line of the sliding scale coincides with the zero line of the main scale and that there is zero distance between the jaws. Examination of the sliding scale reveals that it has ten equally spaced divisions but that the total length of the sliding scale is only 9 mm. This means that each division is exactly 0.9 mm in length, or another way of expressing it is that the length of each vernier division is 0.1 mm less than 1 mm in length. Line 1 on the sliding vernier scale is then just 0.1 mm from line 1 on the main scale, line 2 on the sliding vernier scale is just 0.2 mm from line 2 of the main scale, line 3 on the sliding scale is 0.3 mm from line 3 of the main scale, and so on. Therefore, if the scale is moved so that line 1 of the sliding vernier scale is coinciding with line 1 of the main scale, the jaws of the caliper must be open just 0.1 mm. If the jaws are opened a bit farther, so that line 2 of the sliding vernier scale is coinciding with line 2 of the main scale, then the jaws must be open just 0.2 mm; when line 3 coincides, it is open 0.3 mm, and so on. The object to be measured is placed between the jaws of the caliper. Assume (as typical) that the zero line of the vernier scale does not coincide with a main scale division. For example, assume that the object is of such a length that the sliding vernier scale takes a position which indicates that the object is a little over 4 mm in length. Therefore, the zero line of the sliding scale has a position a little over the exact 4 mm line of the main scale. Just how much it exceeds 4 mm in length will be accurately obtained by applying the knowledge gained from the previous examination of the vernier scale. Suppose that line 1 of the vernier scale is coincident with a main scale division. Therefore, it must be just 0.1 mm over 4 mm in length. The reading would be 4.1 mm.



(a) English Scale



(b) Metric Scale

Figure 1. English/Metric Vernier Calipers

From the preceding description and example of the vernier principle, it should be clear that the method of reading a vernier is to determine the approximate length of an object by noting the position of the zero line of the sliding vernier scale on the main scale; then, if this zero line does not coincide evenly with a main scale division, the exact reading of the length of the object may be obtained by adding to the main scale reading a number of tenths equal to the number of the line of the vernier scale which corresponds with any line of the main scale.

The foregoing description has pertained to the metric scale type of vernier. The English scale type of vernier reference is similar to the metric type, except that the vernier is divided into eight spaces to a corresponding seven on the scale. Thus, it is apparent that the spaces on the vernier are 1/8th of a space shorter than those on the scale. Since a scale space is 1/16 inch, it follows that the vernier spaces are 1/16 x 1/8 or 1/12 inch shorter.

b. Optical Comparator

The optical comparator (Figure 2) is a composite measuring device. The object to be measured is mounted on a stage which is movable in two dimensions. The primary purpose of this instrument is to determine the relative location of two points on the object by bringing one point into the position previously occupied by the other and noting the required displacement of the stage. Coincidence of the two positions is determined by the shadow of the object which is enlarged and projected on a translucent screen provided with index lines. Angles may be measured by rotating the screen and index lines in the optical comparator.

The optical comparator may also be used to compare the projected outline with a translucent drawing of the same object mounted on the projection screen. Coordinate layout and point to point measuring demands precision table travel in two perpendicular planes, accurate graduated screen ring and vernier, and accurate devices to measure the displacement of the table. Further, straight comparison with a tolerance outline chart requires precise optics with exact magnification over all the screen. Regardless of significant accuracies of the combined optical and mechanical system, the final link between the machine and the operator's brain is the human eye, which varies in performance.

One characteristic of the human eye is that the iris expands or contracts to accommodate for different average intensity of light entering the eye. When viewing screen images at extremely high magnification or reflection-formed images, where object reflectivity is poor, the level of light on the screen may be lower than in the room work area. In this event, the operator should wait several seconds before attempting to take readings. The use of curtains and a visor helps stabilize this condition.

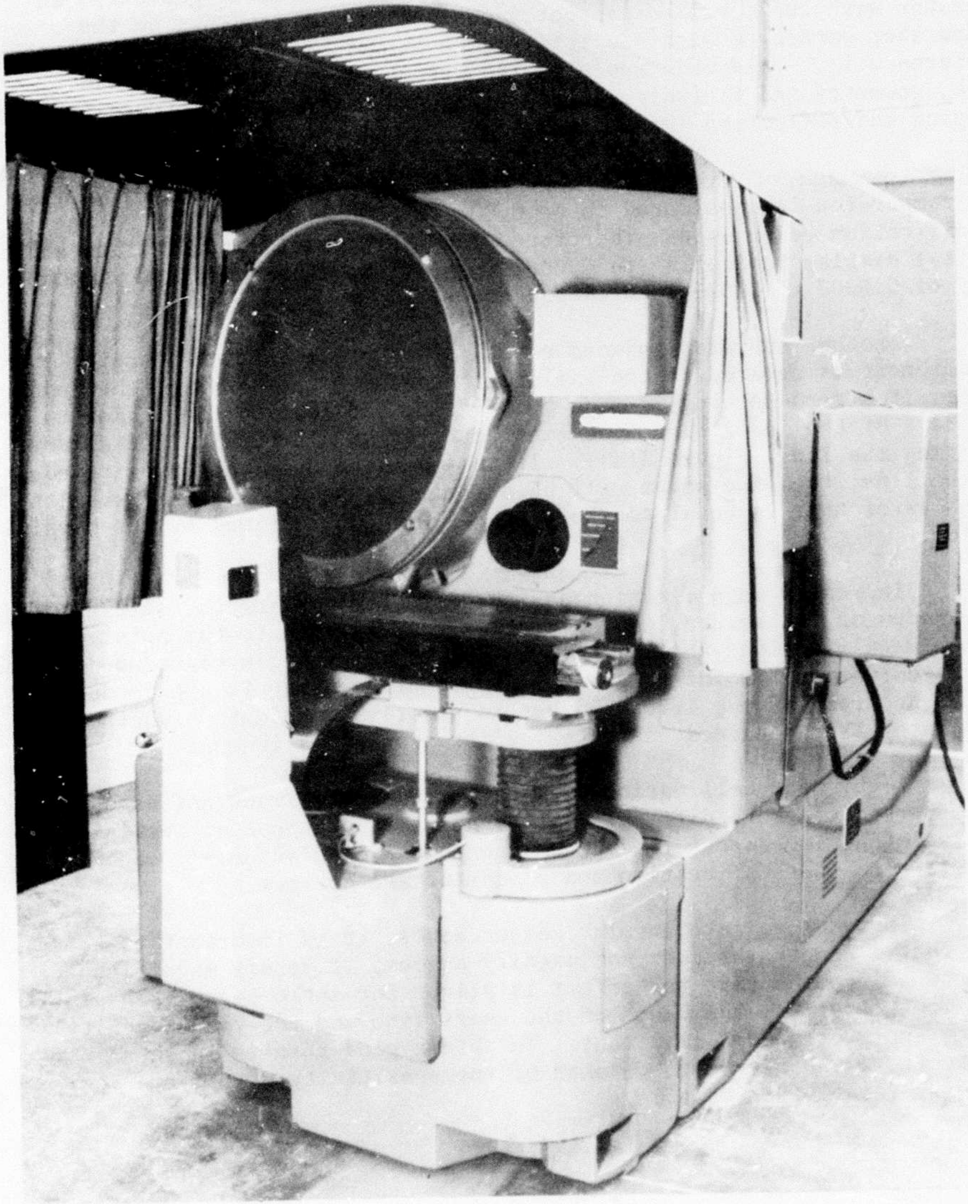


Figure 2. Optical Comparator

Under average viewing conditions, meaning good illumination, with no glare or harsh backlighting and with reasonable object geometry, an operator with 20/20 vision can get repeatability of readings on the comparator screen to within a total range of variation of 0.05 to 0.13 mm (Reference 1). This wide range is the result of variations in object size, geometry and finish, magnification, and differences between projection and reflection image formation.

Because of this limitation, the accuracy of measurement made on the comparator is considered to be 0.002 to 0.005 inch divided by the magnification being used. Therefore, for a ten times magnification and a digital display indicating to 0.0001 inch, there is a variation in accuracy of 0.0002 to 0.005 inch (Reference 1).

Another operator-induced error during precise point-to-point measurement is related to the failure to consider the width of the index line. If a zero setting is made by bringing the edge of the shadow to the edge of the index line, the final setting should be with the shadow covering the line (Figure 3(a)). If the operator uses the opposite edge of the line, then the point will be passed (Figure 3(b)). This results in an error that is equal to the width of the line divided by the magnification.

Increased magnification has the effect of making the setting of the shadow on the screen less critical. This is true to the point where higher magnifications and object size and geometry begin to reduce contrast and illumination. Point-to-point measurement accuracy is not dependent on the accuracy of magnification but is greatly dependent on whether the comparator is maintained in proper calibration and alignment.

Light beam collimation becomes increasingly important as object geometry becomes more difficult. Preventive maintenance at regular intervals should be performed according to the manufacturer's manual to determine that calibration and alignment are correct.

In order to make angular measurements, it is important to keep in mind that a comparator does not magnify angles; it merely magnifies leg length. Since the sine bar effect is slight for short legs, even at high magnification, the width of the chart line and the visual acuity of the operator become important. To obtain good results in these instances, advantage must be taken of the sensitivity of the eye to symmetry rather than to size.

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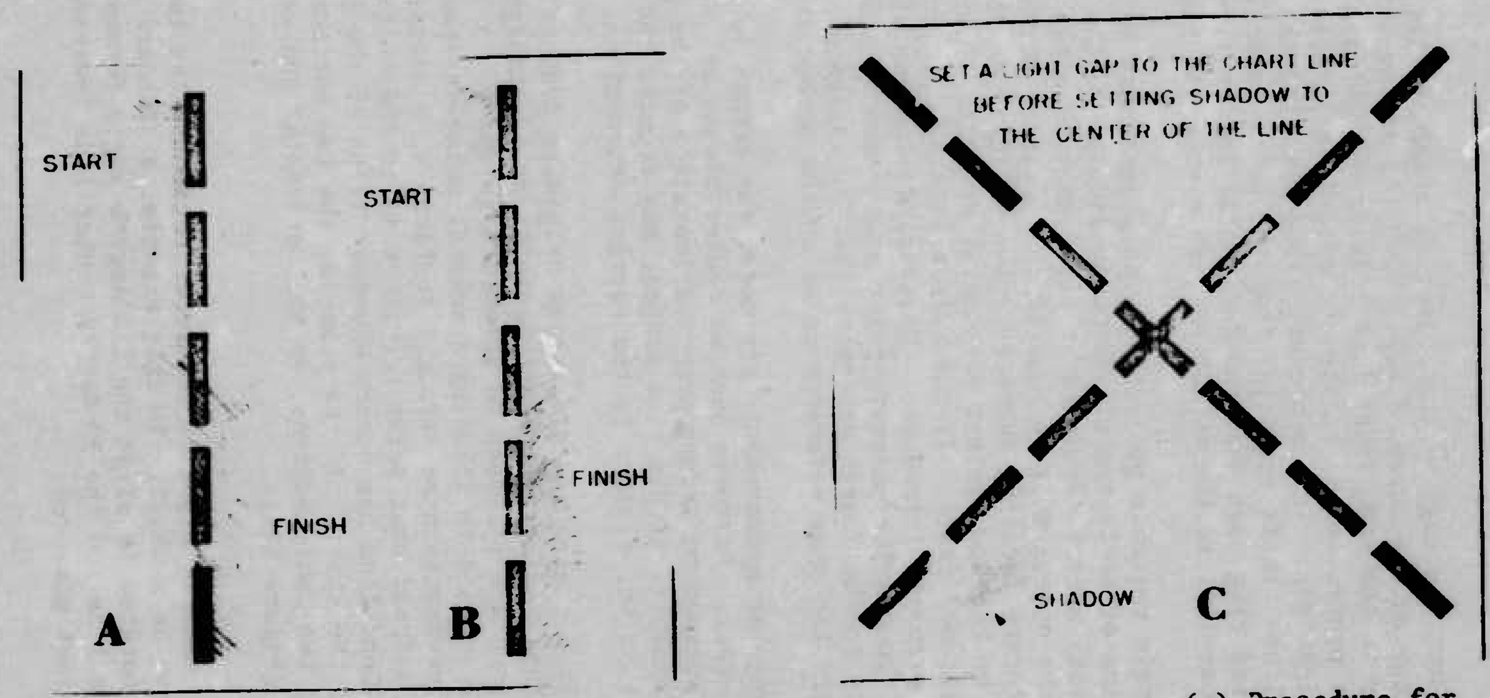
Reference:
1. Fundamental Optical Comparator Techniques. Jones and Lamson Handbook for EPIC-30 Optical Comparator and Measuring Machine.




(a) Correct setting for point-to-point measurement

(b) Erroneous setting for point-to-point measurement

(c) Procedure for angular measurement

Figure 3. Overlay Index Lines

To start, the edge of the object is staged at the intersection of the center cross on the screen lines. Each leg of the angle is brought in turn nearly to the same index line leaving a light gap (Figure 3(c)), and the screen protractor is rotated to establish parallelism before closing the light gap. This procedure avoids burying the angle in the width of the line. It is essential that the chart be positioned correctly in the graduated ring such that the horizontal center line of the screen is optically parallel to the top of the table when the ring is set to zero.

To obtain reliable projections, an object must be staged parallel to the direction of table travel and perpendicular to the optical axis. Since the optical axis is designed at right angles to the table, this simply means the object must be squared on a fixture which provides acceptably accurate locating surfaces. The validity of the staging should always be checked against horizontal or vertical lines on a correctly positioned chart. If the object is staged at an angle, a trigonometric error is introduced. However, deviations of less than one degree can be ignored for point-to-point measurement since sines and tangents are alike to the fourth decimal place for angles of less than one degree, provided the same reference point on the screen is used.

When angular measurements are made, any errors in staging will affect the accuracy. Fixtures such as center blocks and vee blocks are ground and/or scraped to assure proper orientation of the part. Some fixtures, for example, simple vise stages, may require the operator to adjust the part in the fixture to the correct orientation.

Very often angles and dimensions originate from points referenced in drawings. These points involve intersections of straight surfaces in a single plane and are referred to as points in space. A dovetail form tool (Figure 4) is used to illustrate several point-in-space conditions involving the intersection of straight surfaces in a single plane. For example, it is assumed that point 1 (Figure 4) is the origin of the measurements. Condition one occurs whenever lines of the correct included angle appear on the chart. In the example, the two surfaces happen to be parallel to the two table motions, so the horizontal and vertical chart lines are used (Figure 5(a)).

Another example might be the intersection of the two sides of a 120 degree groove in a shaft. In this example, a standard chart (Figure 5(b)) could be rotated to align the 120 degree angle formed by the 30 degree/60 degree line and the 90 degree center line intersection to the image of the object measured.

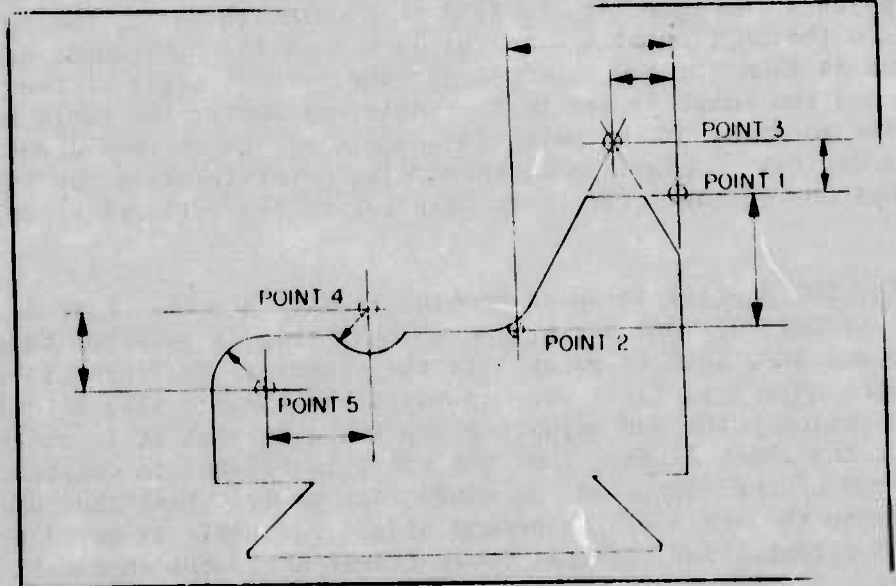


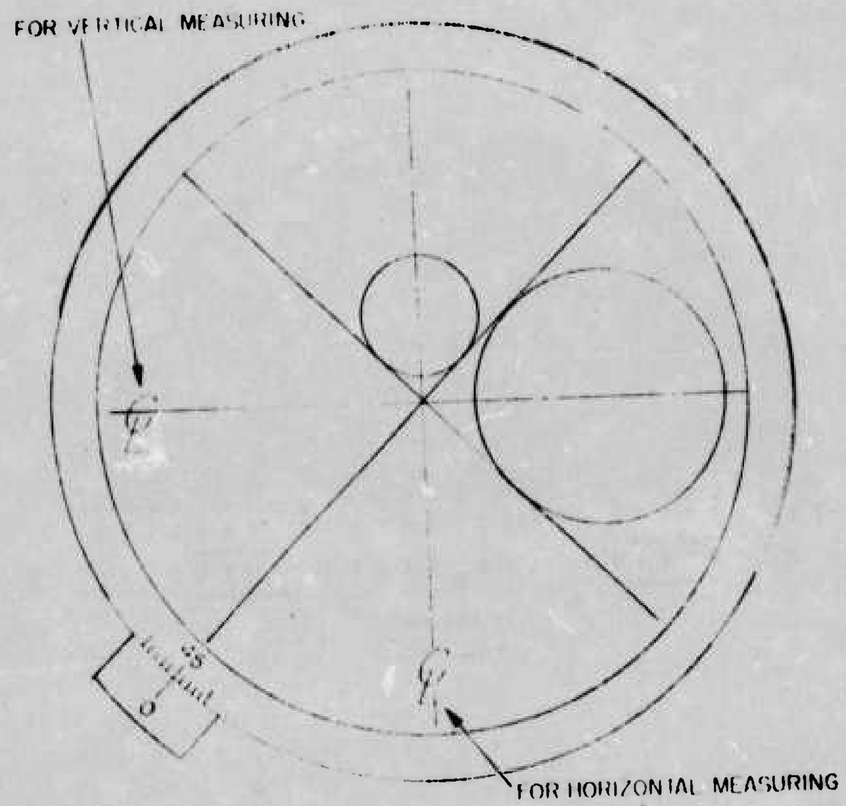

Figure 4. Dovetail Form Tool

In either example, if the origin of the measurement is the point-in-space that has been found, the table digital display is zeroed. If the measurement is being made to that point, the digital display is read to determine the measurement from the origin. The use of digital display reduces the likelihood of operator error.

The second condition of point-in-space involving straight surfaces occurs when one of the surfaces is parallel to one of the table motions, such as point 2 in Figure 4. To find this point-in-space, the surface parallel to the horizontal tab travel is set to the horizontal center line. The screen is then rotated to construct the correct angle of the second surface, and the image is set to the angle, by moving the table horizontally. The location of the point-in-space may then be read directly from the table digital display. A corresponding point-in-space condition occurs when one of the surfaces is parallel to the vertical travel of the table.

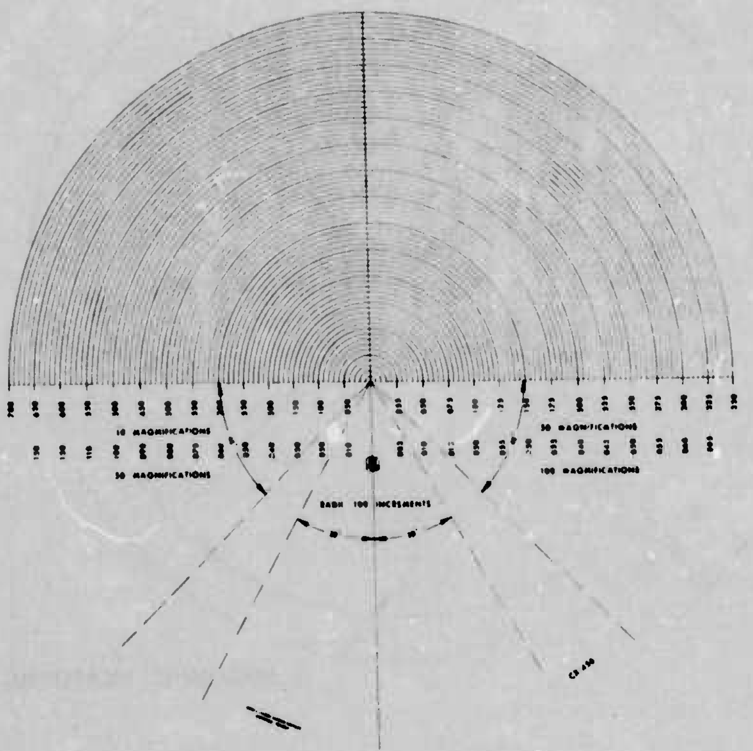
The third point-in-space problem occurs when the chart does not have the correct included angle, and neither side is parallel to one of the table motions, such as point 3 in the example. To find this point in space the object is first roughly positioned by mentally visualizing the intersection point and adjusting the table so that it is at the intersection of the chart lines. Then the chart is rotated to construct the angle of one of the sides and the shadow set to it. Next the chart frame is rotated to the angle of the second side. The table is moved using both the horizontal and vertical table travel until the shadow is superimposed over the line. If the two sides have the same angle, half the adjustment to close the gap is taken horizontally and half vertically. If the angles for the sides are not equal, then greater adjustment will be required on one motion than the other. Next, the angle of the first side is reconstructed, and again the gap is closed, using both of the table travels. Each time, the alternate side is constructed, and the shadow is set to it using both table travels. The error of correct position is then reduced until the shadow is superimposed on the chart lines at each side, without moving the table. When this occurs, the coordinate location of the point may be read directly from the table digital display and is located at the intersection of chart lines.

The coordinate locations of the center points of radii or holes, such as points 4 and 5 in the example, may be quickly determined using a radius chart. The shadow of the first radius is set concentric with one of the lines on the radius chart. The size of the radius itself is read from the scale on the radius chart and the location of the center of radius is read from table digital displays.




(a) Standard Chart

Figure 5. Comparator Chart Overlay




(b) Chart as used for 120 degree angle

Figure 5. Comparator Chart Overlay (Concluded)

The center distance of small holes can also be measured by using the perpendicular lines on the chart to form an optical vee block as shown in Figure 5(a). The chart is rotated to exactly 45 degrees to form a 90 degree vee which has center lines that are optically perpendicular to the table travels. The shadow of the hole is "laid" into the vee, and the center distance is read from the table digital display when the second hole or other point on the object is set to the chart lines. When the hole center location is measured, the holes do not have to be the same size.

Practically all of the techniques discussed can be accomplished with the charts shown in Figures 5(a) and 5(b). A well maintained optical and mechanical system is essential. Ingenuity is always a help, and if a picture of the problem can be drawn, the chart and comparator can find the answer.

## 2. WEIGHT (MASS)

Mass is sometimes defined as the quantity of matter in an object, and the value for Mass (of a given object) is constant no matter where in the universe the measurement is obtained. This definition, however, does not furnish a basis for operations by which various masses can be measured or compared. True, a body can be weighed with a spring balance, and the more massive it is, the more it will weigh; but the weight will vary from point-to-point on the earth's surface. Hence, merely weighing a body does not uniquely determine the Mass.

However, for many earth-bound engineering applications, the changes in weight from location to location are insignificant, and therefore, the terms weight and mass often are used interchangeably. In many engineering codes and standards, the term weight is used even though this weight is determined by the use of masses.

Since all weighing scales are calibrated by the use of standard weights, the act of weighing does not measure the weight force as defined by the physicist, i.e., the pull of gravity. If all scales are calibrated at the point of use by standard masses and are corrected for the buoyancy of the air if necessary, their readings are independent of local gravity.

The National Bureau of Standards lists the tolerance limits for several classes of scale and balance weights for the laboratory. Acceptance tolerances are on the order of 1 part in 100,000 for precision balance weights. Tolerances vary with the size of the weight. Precision weights must be handled according to strict rules to prevent damage. All weights and objects should be handled with forceps or tweezers, never with the fingers, because oil or moisture from the hands can cause erroneous mass indications. Further, they should be placed as near as possible to the center of the pans.

Small mass standards for use with precision balances are available in denominations from 1 mg to 100 grams. The 100 gram weight is corrected to 0.5 mg or 1 part in 200,000. The 1 mg weight will be correct to 0.01 mg or only to 1 part in 100. The number of weights in the set is sufficient to permit cross-checking on the balances.

Since balances are mechanical devices used to determine the mass of objects and because the mass of the different objects varies, the choice of the balance to be used for any determination will be governed by the total mass of the object and by the sensitivity desired. Therefore, the first decision to be faced is which balance to use. The precision required is the second decision.

All balances are precise and expensive instruments, and extreme care should be used when handling and using them. A variety of balances are found in the Projectile Measurements and Instrumentation Laboratory. These range from a rough measuring device (the triple-beam balance) that is sensitive to 0.1 gram, to the analytical balance that is sensitive to a fraction of a microgram.

The following comprehensive rules should be observed in caring for and using the balances. These are general rules for all balances and some may not be applicable in work with the less sensitive measuring devices.

(a) The balance should be level.

(b) The balance is inspected to be certain that it is working properly; and calibrated, undamaged weights are used.

(c) The balance zero is checked.

(d) The beam is locked before removing or changing weights or objects to be weighed.

(e) The balance is kept scrupulously clean.

(f) To avoid parallax errors, work is performed in front of the balances.

(g) All weights and objects are handled with forceps, never with fingers, for reasons stated previously. The forceps are used to place the weights and objects as near as possible to the center of the pans.

(h) Weighing objects that have not been stabilized to room temperature is to be avoided. If the temperature of an object is different from that of the local environment, condensation can form on the object, thus giving a false mass reading.

(i) The locking mechanism is to be released slowly, avoiding jars.

(j) The balances should <u>never</u> be overloaded.

(k) Moist objects should never be weighted directly on the pans.

(l) The balance case (if a part of the balance) should be closed.

(m) Proper annotation of weights in a notebook or similar record should be made.

a. Triple-Beam Balance

The triple-beam balance (Figure 6) is suitable for weighing relatively light objects (5 pounds or 2610 grams maximum). A heavy duty balance capable of weighing larger items (45 pounds or 20 kg maximum) also works on the same general principle (Figure 7).

Before any weighing is attempted, the balance should be placed upon a reasonably flat and level surface. The beam should be near a zero balance with all poises at zero and the tare poise against its stop. A final zero balance is attained by means of the knurled zero adjust knob at the left end of the beam. It is advisable to check the zero balance periodically, since foreign material may accumulate on the plate or beams and cause a slight change in the balance position. Whenever the balance is moved, the zero balance must be rechecked, since it will be affected by a change in the inclination of the working surface.

The determination of the weight of any object on the triple-beam balance is a very simple and rapid operation. After obtaining a zero balance, the object to be weighed is placed on the load receiving platform. Then the center poise is moved to the first notch where it causes the beam pointer to drop, then moved back one notch so that the pointer will rise. The next step is to manipulate the rear poise in the same manner. The front poise is then slid to the position which brings the beam into balance. The weight of the object can then be directly read by adding the values indicated by the poises. Magnetic damping helps bring the beam to rest quickly.

The capacity of the triple-beam balance is extended beyond graduated beam capacity by means of attachment weights available as accessories. The attachment weights are suspended from the pivots on the end of the beam and extend the capacities of metric models to 2610 grams to to 20 kg depending on the model.

Figure 6. Triple-Beam Balance (2160 grams)

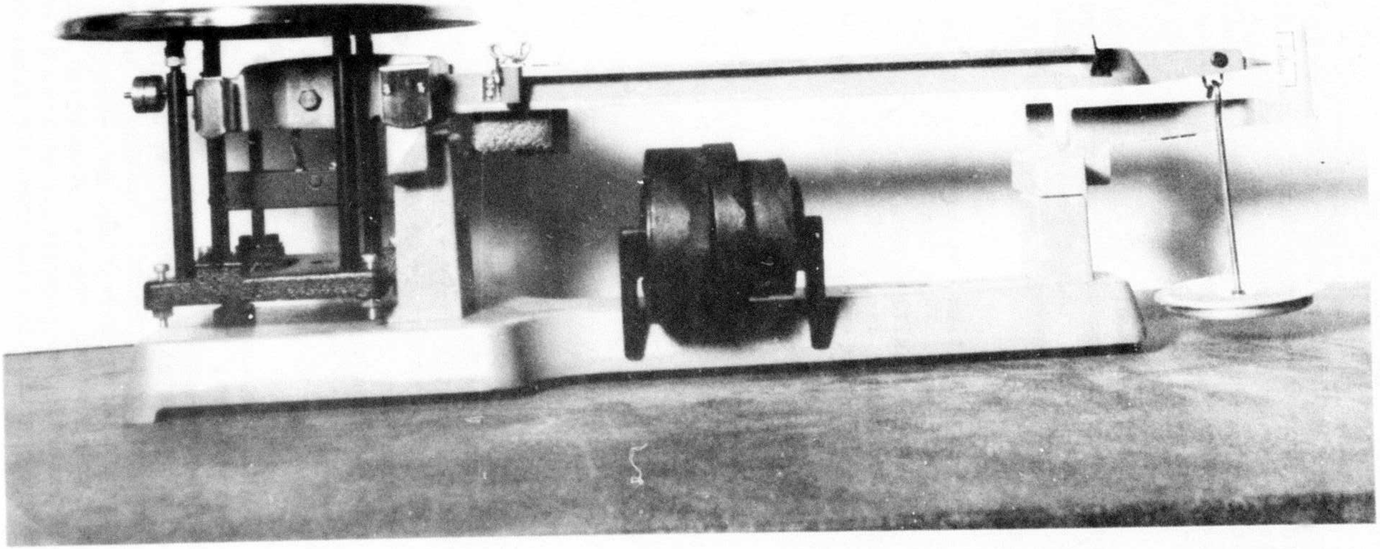


Figure 7.  Heavy-Duty 20 kg Balance Scales

When the balance is not in use, remove any load from the pan and the weights from the balance hanger, and slide one of the poises out so the balance is not in equilibrium. This will prevent the balance from oscillating while not in use and thus prevent unnecessary wear. Also, the balance should be stored in a place where vibrations will not be transmitted to the bearings and knives, causing undue wear.

The balance should be kept clean at all times, with particular attention to preventing dirt from accumulating in the vicinity of the bearings. Oil or any other type lubricant should never be applied to the knife edges or bearings; such lubrication inhibits the working of the balance.

From time to time, it is necessary to clean accumulated debris from the magnet face. This can best be accomplished by inserting a piece of pressure sensitive adhesive tape into the magnet slot and pressing it against the magnet face. This will pick up attracted material and prevent it from interfering with movement of the damper vane.

b. Top Loading Balance

Top loading balances (Figures 8 and 9) are the easiest of all mechanical balances to use, because an object can be placed on the pan and the results can be read out digitally directly from a dial. Top loaders also offer easy taring. The adapter or container is placed on the balance pan, the tare knob is backed to zero, and the weight of the adapter is eliminated. Only the mass of the object is read from the dial.

The basic principle of operation of the top loading balance is the same as that of the unequal-arm balance, that is, by substitution. The balance consists of an asymmetric beam. The maximum load is placed on both sides. On the shorter end are a pan and a full complement of weights. A counterweight is used on the longer end to impart equilibrium to the system. When a load is placed on the pan, an equivalent weight must be removed from the load side within the range of the optical scale to bring the balance into equilibrium. The total weights removed, plus the optical-scale reading, equal the weight on the pan.

The basic controls on most single pan analytical balances (Figures 8 and 9) may vary with the manufacturer. In general, a pan arrestment control assures constant position of the beam between and during weighing and also protects the bearing surface from excessive wear and injury due to shocks. Another (three-position) arrestment control may be used to arrest the pan when removing or placing objects on the pan, when the balance is being moved and when the balance is not in use. A partial arrest position is used to obtain preliminary balance, and a release position is used when the final weighing is being made.

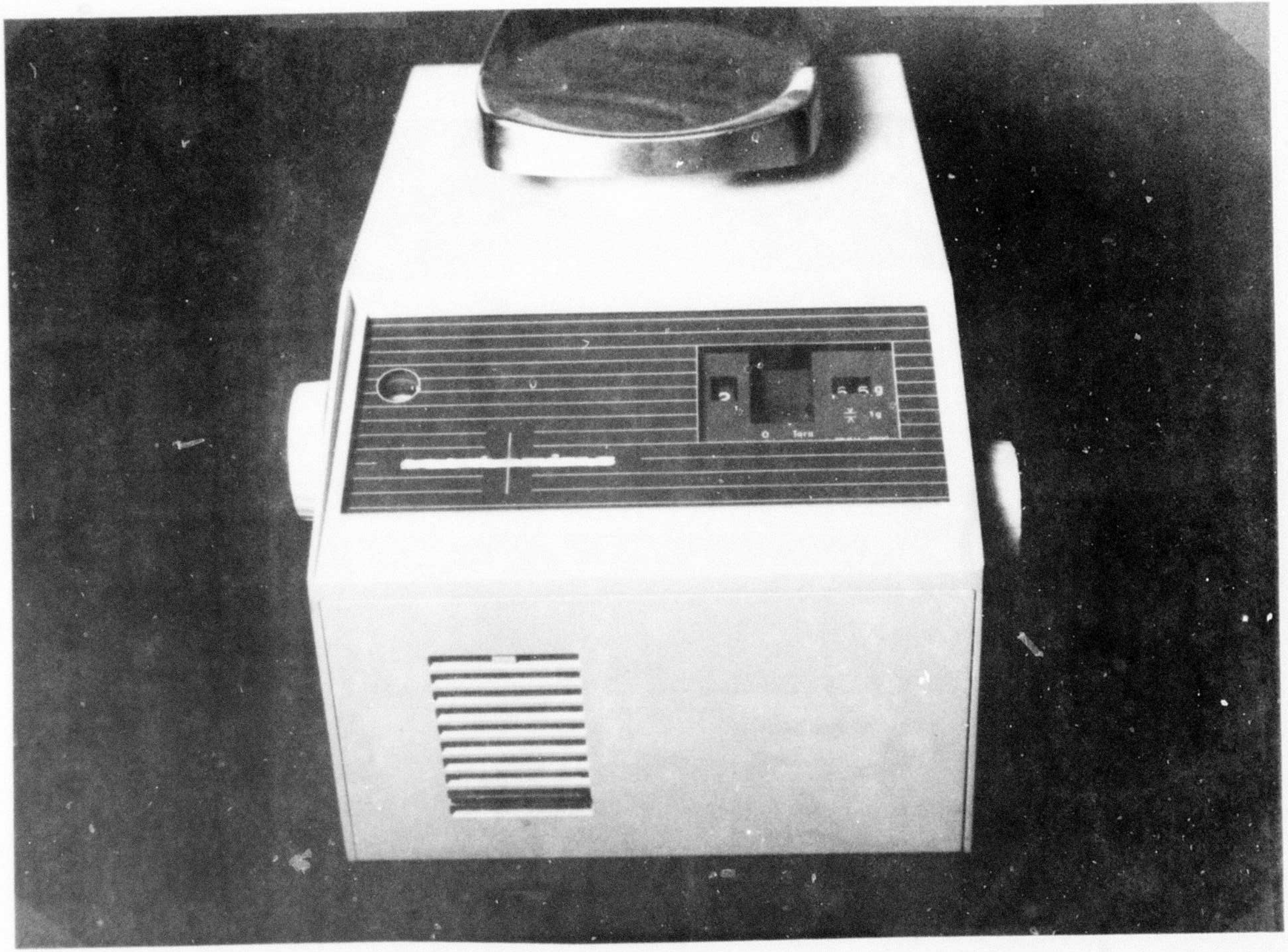


Figure 8. Precision Balance (1000 grams)

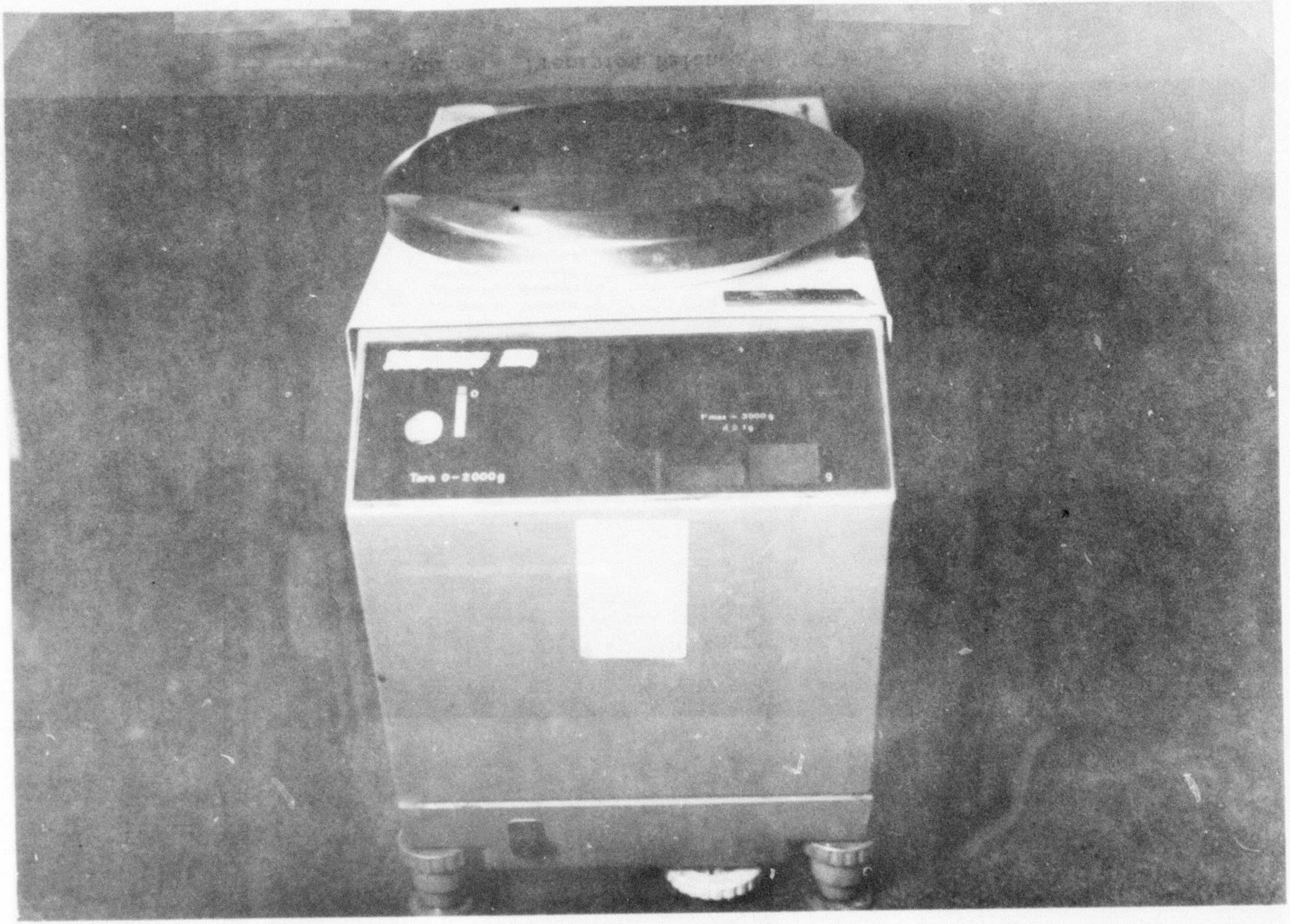



Figure 9. Precision Balance (3000 grams)

A zero-adjust knob positions the optical scale to read zero when the pan is empty because of minute changes in beam position. Other weight-setting knobs remove and replace weights from the beam. The optical scale adjustment positions the optical scale relative to a reference line so that the final mass can be obtained.

Some fundamental rules should be followed for maintaining accuracy and reproducibility of balances. The balance is set up where it will be free from vibration, preferably in a room with constant temperature and humidity. A table of adequate weight and stability should be selected for a base. The balance is to be loaded and unloaded only when it is arrested. When arrested, the beam and suspension rest on arrestment pins. The knife-edges are thus fully unweighted and not liable to damage by sudden load changes such as are caused by loading or removing the object to be weighed. The arrestment lever (knob) should be moved slowly until the scale illumination has been switched on. Weight-setting knobs should only be turned when the balance is semi-released. Partial release protects the knife edges from damage during weight setting; however, not all models have this feature. Abrupt loading changes resulting from applying and removing the weights are to a large extent absorbed by the arrestment pins as, when semi-released, the beam movement is limited.

Weighing requires the following operations:

(a) Checking the level. The spirit level is checked to ensure that the balance is properly leveled. If necessary, the balance is re-leveled.

(b) Checking the zero point. The pan is unloaded and cleaned if necessary. The side window is closed. All weights are set to zero. The micrometer scale is set to zero. The balance is released fully. When the projected scale is at rest, the zero line of the scale is brought to read zero. The balance is then arrested.

(c) Placing an object to be weighed on the balance. The object to be weighed should be placed on the balance only in the arrested state. Whenever possible, a pair of tweezers should be used to avoid humidity and heat being transmitted by the hands to the objects to be weighed and to the inside of the weighing chamber.

(d) Setting of weight. The balance is semi-released. The optical scale moves downward several graduations. The highest weight-setting knob is turned. When the scale moves upward, the knob is turned back one step. The same operation is repeated with the remaining weight setting knobs. The balance is arrested after having set the weights. In this position, the pan becomes operative. The balance is released fully and allowed to come to rest.

3. CENTER OF GRAVITY

The center of gravity of a solid object is an imaginary point at which all its weight may be considered to be concentrated or the point through which the resultant weight passes. This definition implies that the forces of gravity acting upon the different parts of an object are all directed downward (relatively speaking) toward the earth. The forces of gravity acting upon the object thus form a system of parallel forces. Using the basic sum of the forces equation from vector analysis, the resultant of these forces can be found as a single force called the weight of the object. This force acts through the center of gravity of the object. The center of gravity can be theoretically located for various symmetrical, solid objects, and is the basis used for constructing solid objects to be used as equipment calibration models.

A solid object is said to be symmetrical with respect to a plane if that plane divides every line passing through the solid at right angles to the plane into two segments of equal length. Such a plane is called a plane of symmetry. If an object has a plane of symmetry and is constructed of homogeneous material, the center of gravity of the object must lie in that plane. If an object is symmetrical with respect to two planes, its center of gravity must lie on the intersection line of those two planes. If there are three planes of symmetry at right angles to each other, the center of gravity of the object is at the point at which these three planes intersect.

Assuming that some homogeneous material can be acquired (material with constant density per unit volume), a number of simple symmetrical objects may be fabricated for calibration models:

(a) A sphere is symmetrical with respect to every plane that passes through the center of the sphere. The center of gravity of a sphere, therefore, must be at its center.

(b) A hemisphere is obtained by cutting a sphere into two equal parts along any plane through the center. The center of gravity of a hemisphere lies on a line that passes through the center of the original sphere and is perpendicular to the cutting plane. The perpendicular distance from the cutting plane to the center of gravity of the hemisphere is 3/8 the radius of the sphere.

(c) A right cylinder or a rectangular object is symmetrical with respect to a plane that is parallel to the planes of the bases and lies midway between them. Such an object is also symmetrical with respect to the planes that pass through axes of symmetry in the bases. The center of gravity of a right cylinder or a rectangular object lies on the axis that joins the centers of the bases, and is midway between the bases.

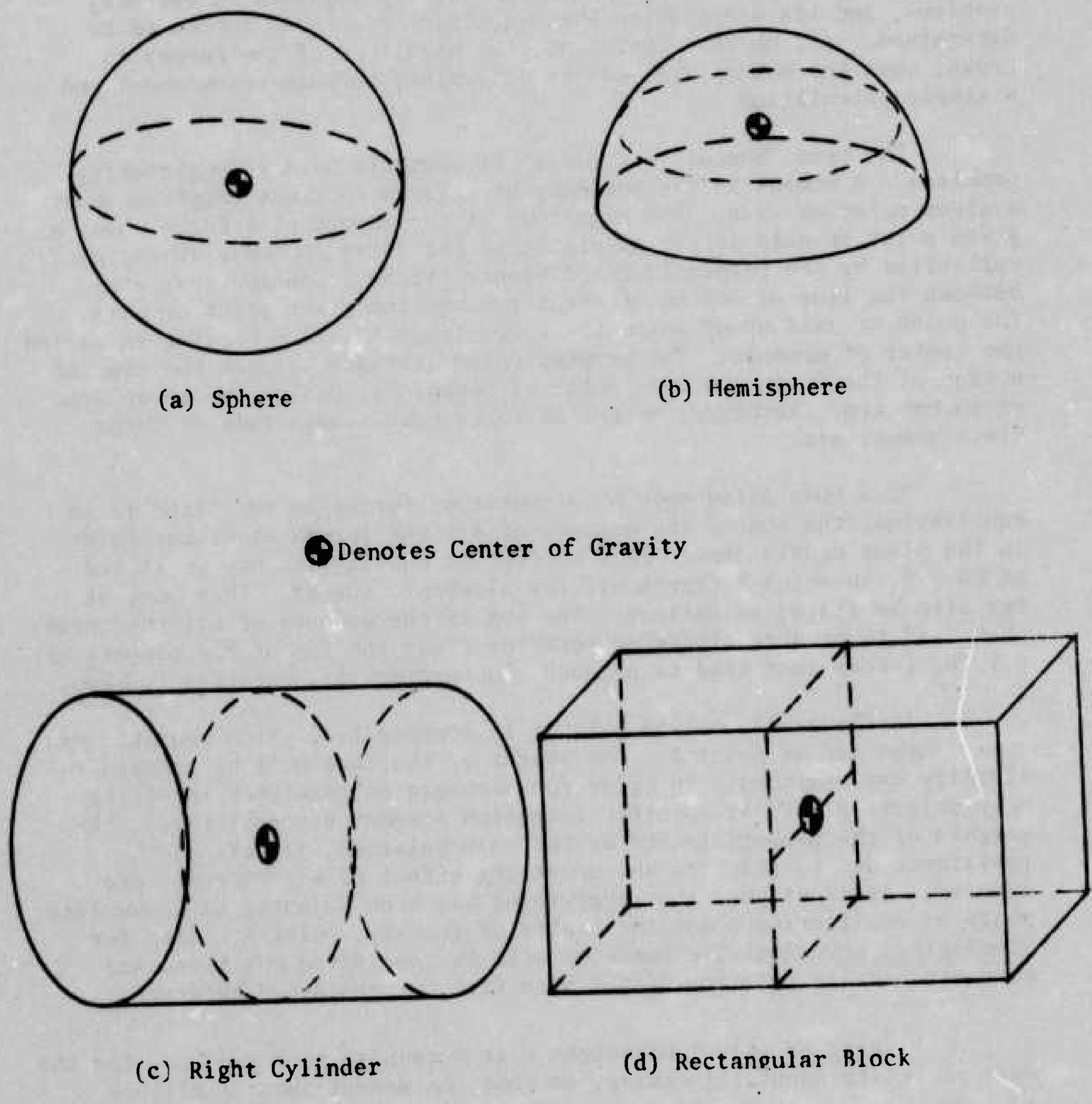



Figure 10. Calibration Models

A graphic diagram of the calibration models is shown in Figure 10.

The principle of moments is frequently employed in everyday problems, and its use enables the magnitude of unknown forces to be determined. If, on the other hand, the magnitude of the forces is known, then the moment arms can be determined through measurement and a simple calculation.

The term "moment of a force" is commonly used in engineering problems. A moment is the tendency of a force to cause rotation about a given point or axis. The magnitude of the moment of a force about a given point or axis is the magnitude of the force (pounds, dynes, etc) multiplied by the perpendicular distance (inches, centimeters, etc) between the line of action of the force and the given point or axis. The point or axis about which the force tends to cause turning is called the center of moments. The perpendicular distance between the line of action of the force and the center of moments is called the lever arm, or moment arm; therefore, moment of force equals magnitude of force times moment arm.

If a body acted upon by a number of forces in one plane is in equilibrium, the sum of the moments of all the forces about any point in the plane equals zero. This concept of equilibrium may be stated as $\Sigma M = 0$, in which $\Sigma$ represents the algebraic sum of. This concept may also be stated as follows: The sum of the moments of all the forces that tend to produce clockwise rotation minus the sum of the moments of all the forces that tend to produce counterclockwise rotation is zero.

In Figure 11, a beam balance is represented, which consists of a bar supported at point A. The weight of the beam will be ignored to simplify explanations. In order for accurate calculations involving only objects placed at specific locations (moment arm positions), the weights of the members (parts of the beam balance), as well as the resistance due to friction and upsetting effect of air current, are ignored. It is assumed that everything has been adjusted to a complete state of equilibrium about the center of gravity, point A. Also for simplicity, any clockwise tendency will be considered positive, and counterclockwise rotation of the beam will be considered negative.

Placing an object of weight w at B results in a tendency for the beam to rotate counterclockwise, so that the moment about A will be $-wL$ or

$$M_B = -wL. \tag{5}$$

Now, if a comparable weight is placed at C at a point the same distance L from point A, there will be a clockwise rotation or positive moment that is equal in magnitude to moment $M_B$. Therefore,

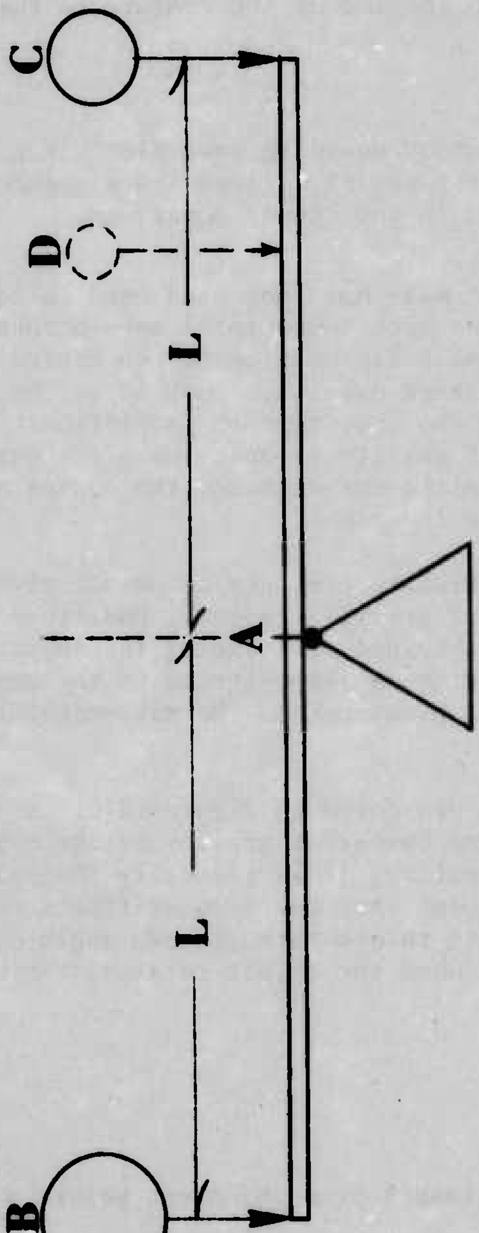



Figure 11. Equal Arm Beam Balance

$$\Sigma M = M_C + M_B = wL - wL = 0 \qquad (6)$$

Equation (6) could be rewritten such that the sum of the moments to the left of A is equivalent to the sum of the moments to the right of A. In essence:

$$\Sigma M_L = \Sigma M_R. \qquad (7)$$

This implies that the equation would be equivalent if a force at D times its distance from A plus a force at C times its distance from A is equal to the moment $M_C$ generated in the former equations.

The balancing of a mass has long been used to determine the center of gravity location and has been found to be more accurate than other methods, because a very small dislocation of the center of mass from the pivot axis shows up as a large unbalance quantity. The limitations in this technique arise from the fact that it is difficult to shift large masses until the center of gravity is over the pivot point. Also, the sensitivity is proportional to the distance the center of gravity of the mass is above or below the pivot.

Figure 12(a) graphically presents an object pivoted as an inverted pendulum with the center of gravity precisely positioned above the pivot. The balance condition is obtained by shifting the object to the left or right. The center of gravity can be referred to the nose or front of the object with respect to the pivot point. No mathematical manipulation is required.

The configuration presented in Figure 12(b) is especially sensitive to the position of the center of gravity of the object with respect to the pivot point. In reality, it is virtually impossible to balance a vehicle above a pivot point that has zero stiffness and zero friction because the object tends to rotate through some angle $\theta$, as shown in the Figure 12(b). Therefore, when the object rotates, the overturning moment M can be written:

$$M = WY \sin \theta \qquad (8)$$

where:

Y = the vertical length from the pivot point to the center of gravity.

$\theta$ = the angle rotated.

Thus, whenever the center of gravity is above the pivot point the system is unstable. The system could be stabilized by putting the center of gravity below the pivot point (a matter of swinging weight below the system pivot point). However, that would reduce the sensitivity and

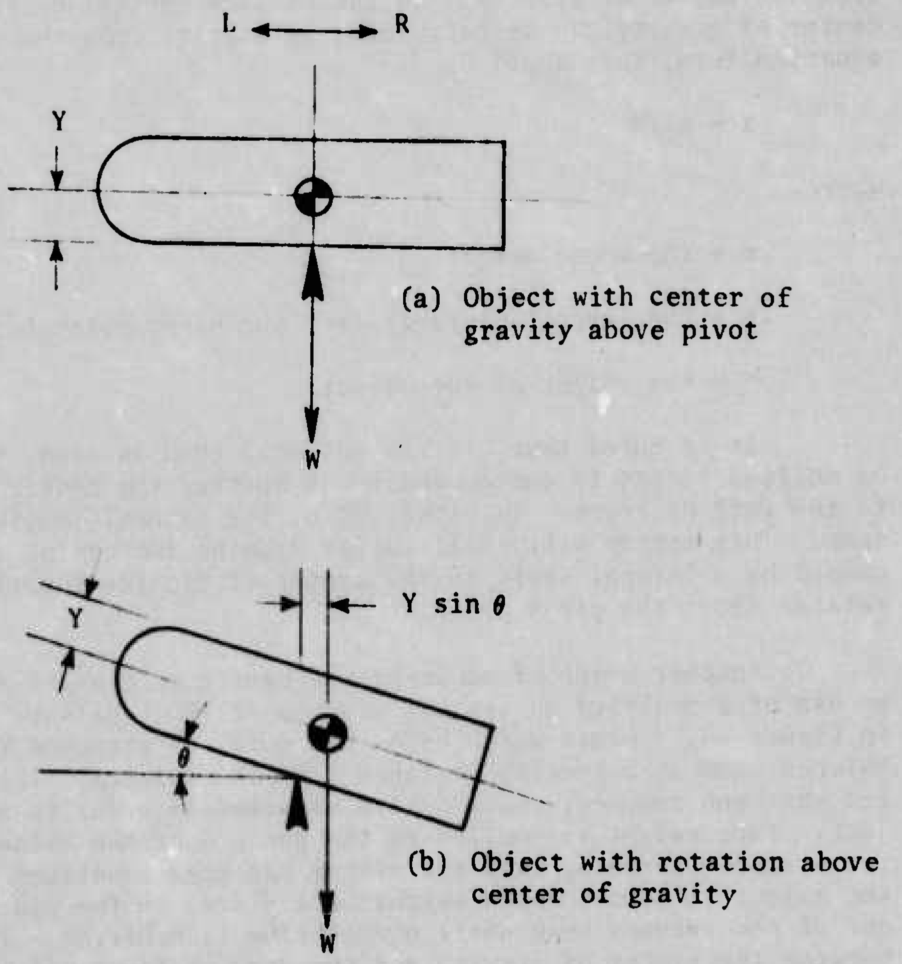



Figure 12. Object Suspended as Inverted Pendulum (unstable equilibrium condition.)

accuracy, since the restoring forces due to gravity acting on the balance may become so large that variations of position and mass can be made along the beam without changing the equilibrium point.

The system using the inverted pendulum characteristics and balancing the object using the balanced beam feature discussed earlier could be improved to a workable unit. This would be done by adding a known weight from the object on a frictionless pivot at a specific distance laterally from the center of gravity. In the balance condition (Figure 13), the center of gravity can be calculated as X units from the pivot point. In equation form, this would be

$$X = mL/W \tag{9}$$

where:

$m$ = the added weight

$L$ = the lateral distance from the pivot point to the added weight

$W$ = the weight of the object.

It is noted that, if the balanced beam is used, the weight can be shifted to either end depending on whether the center of gravity lies to the left or right. In either case, the lateral length remains the same. This system will still suffer from an increasing instability caused by a lateral shift in the center of gravity location as the object rotates about the pivot point.

Another means of locating the center of gravity of an object is by use of a modified analytical balance of the equal-arm type. As shown in Figure 14, a plate which holds the model is attached to one end of the balance beam at a precise distance from the fulcrum. The pan on the left end has been removed, and a cradle attached in order to position the object. Tare weight is applied to the pan end of the balance to equalize the system. Assuming that the system has been equalized prior to placing the object in place, known weights are placed in the pan at the opposite end of the balance beam until equilibrium is achieved. The distance between the center of gravity and the nose or front end of the object is then computed using the following equation:

$$\Sigma M = wX_R + W_aX_R - W_oX_L - W_oX_{CG} = 0 \tag{10}$$

or

$$X_{CG} = X_R\left[w + W_a\right]/W_o - X_L. \tag{11}$$

where:

$W_o$ = weight of the object

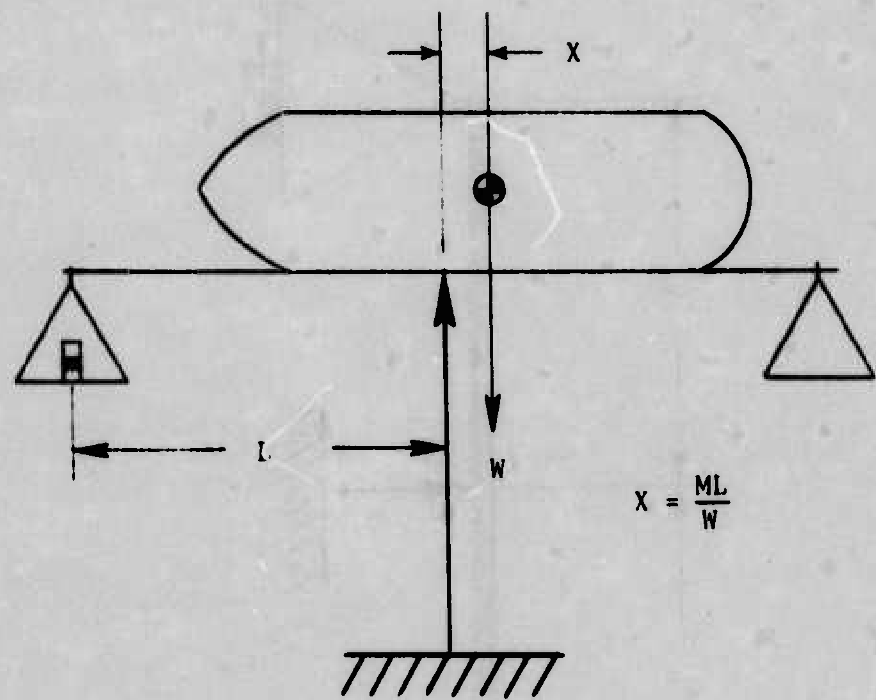



Figure 13. Inverted Pendulum - Balanced Beam Fixture

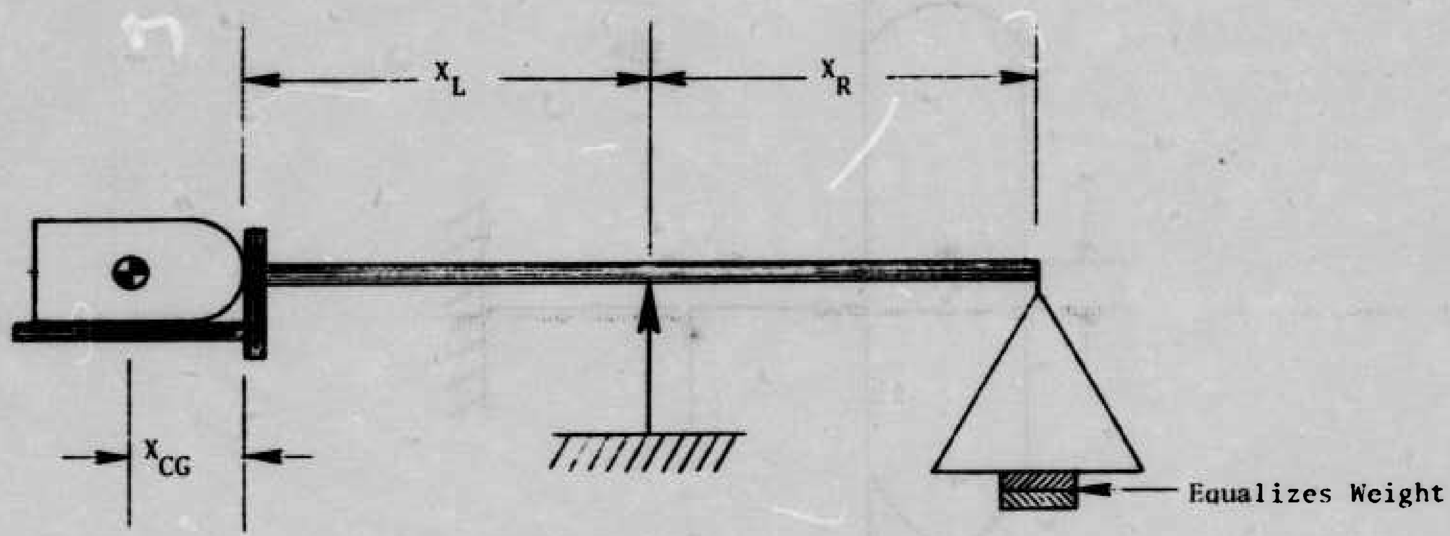



Figure 14. Modified Equal-Arm Balance Moment of Inertia Determiner

$W_a$ = sum of weight added to achieve balance equilibrium

$w$ = system equalizer tare weight without object on cradle

$X_R$ = distance from pivot axis to support point of pan

$X_L$ = distance from pivot axis to contact point of object in cradle

The distances $X_R$ and $X_L$ can be accurately determined by prior calibration of the balance, using one or all of the previously covered calibration objects. By this method, the center of gravity of model can be determined to within 0.025 mm.

The center of gravity balance now in use (Figure 15) consists of an aluminum beam marked in tenths from an attached knife edge, a platform for mounting the test object on the opposite side of the knife edge, and a knife-edge-mounted movable pan for applying weights for balancing. A primary weight is attached to the beam for balancing the empty fixture, and the distance from this weight to the knife edge is adjustable. An additional weight is attached above the fulcrum point to allow adjustment for stability or vertical position of the center of gravity of the empty beam.

Before use, the system should be so balanced that the pointer reads zero. If slightly displaced, the pointer should tend to return to zero. This will be true only if the center of gravity of the system is lower than the support pivot where the knife edges rest. If the center of gravity is higher than the pivot axis, the equilibrium would be unstable, and the beam would keep on turning to the stop.

The sensitivity of the balance depends on how far the center of gravity of the moving system is located below the knife edge pivot. Schematically speaking if the beam is balanced (Figure 16(a)), then the center of gravity G is directly below the axis of support. When this beam is displaced about the pivot point, the center of gravity is effectively displaced to one side (Figure 16(b)). The weight acting through G exerts a restoring torque, tending to move the beam to the zero position. The farther below the pivot point, the larger will be the restoring moment, and consequently, the larger the weight required to move the balance from the zero position. Simply stated, the nearer the center of gravity is to the pivot point, the higher the sensitivity of the balance, since a small weight on the scale pan will produce sufficient torque to turn the beam through a measurable angle. The farther away from the pivot point the center of gravity is located, the less sensitive will be the balance.

Before an object for test is placed on the fixture end of the balance, the platform is adjusted so the vertical position of the center of gravity of the object will be in the same approximate plane as the kife edge and the graduated surface of the beam. In this position, the sensitivity and stability of the balance will not change when the balance is loaded. The counterweight is mounted such that during its range of

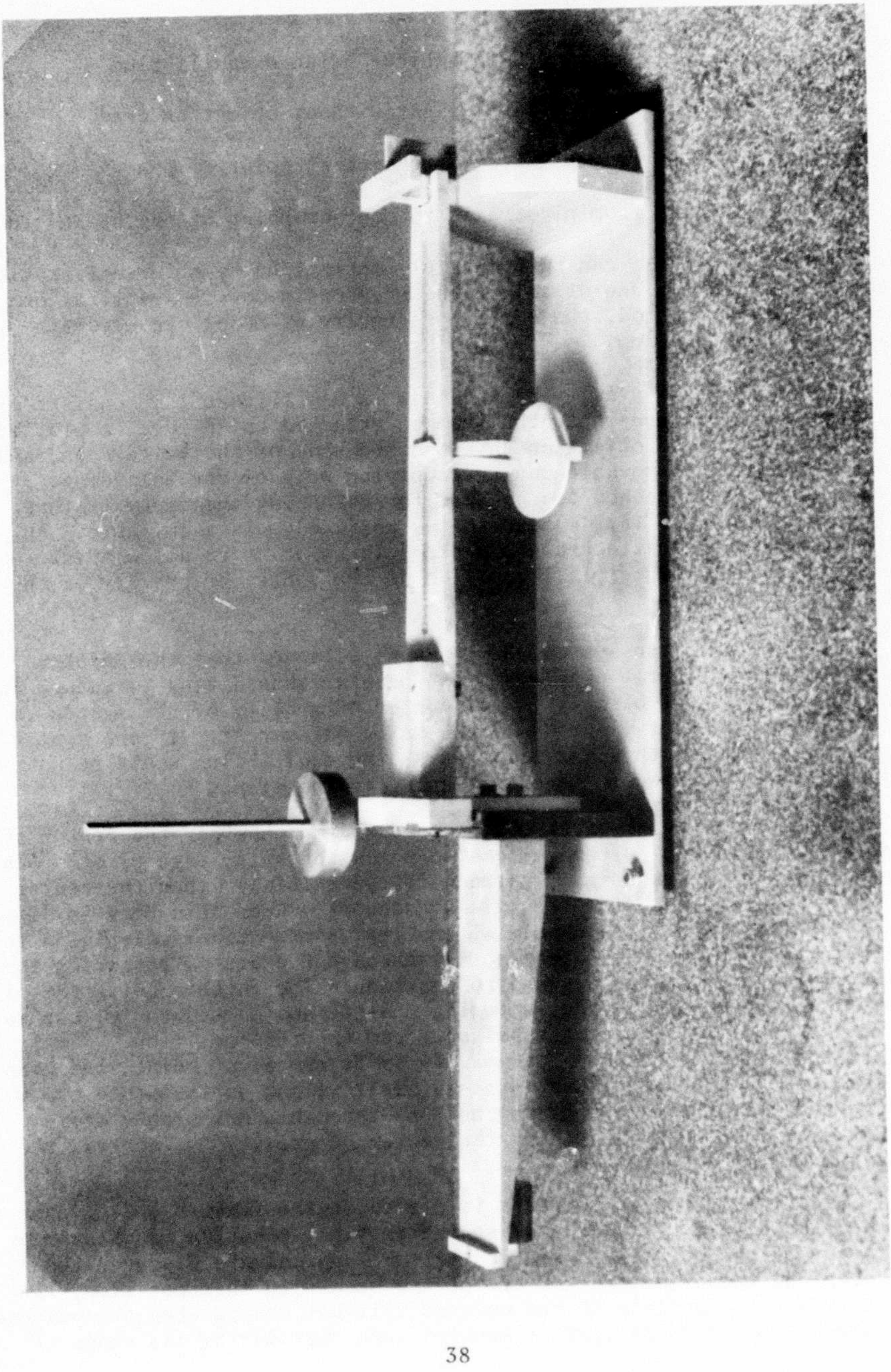


Figure 15. Center of Gravity Balance

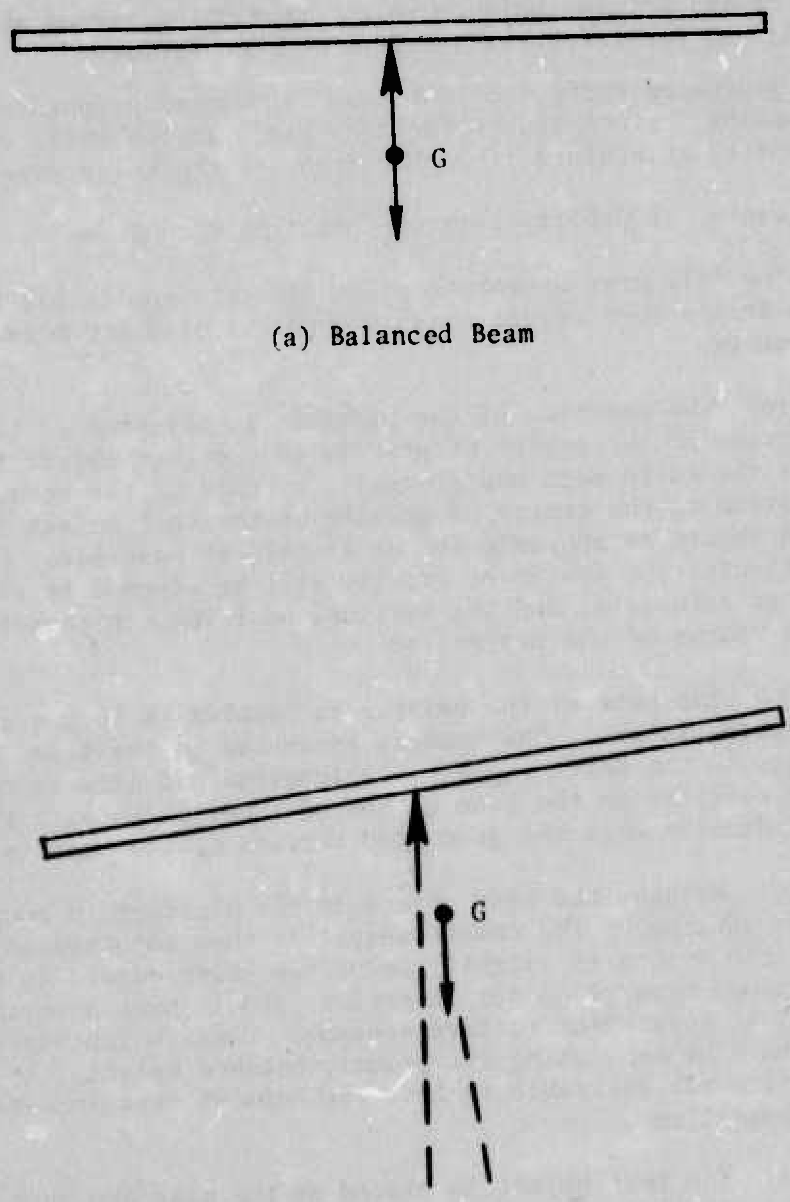



Figure 16. Equilibrium of Balance

adjustment it will move vertically over the knife edge. This prevents large adjustments of the primary balance weight from being needed when the counterweight is changed for different settings of the platform.

The graduated surface of the beam is mounted perpendicular to the vertical mounting surface and in the same plane as the knife edge. This condition must be maintained if resharpening of the knife edge is attempted.

a. Center of Gravity Balance Operating Procedures

The following operating procedures are applicable to any balance beam device (For example, see Figure 15) used for measuring the center of gravity.

(a) The position of the platform is adjusted so that the vertical position of the center of gravity of the test object is in the same plane as the knife edge and graduated surface of the beam. If the vertical position of the center of gravity of the test object is not known, this position should be approximated as closely as possible. For most projectile objects, the center of gravity will be assumed to exist on the longitudinal or spin axis, and the vertical position can therefore be adjusted to the radius of the projectile.

(b) The base of the balance is leveled in both axes, using the three leveling screws. The beam is assembled to the base, using care to avoid damage to the knife edges. The longitudinal axis is realigned such that the hairline on the side of the beam coincides with the zero mark on the indicator when the graduated surface of the beam is level.

(c) Without the test object on the platform, a rough balance of the beam is obtained. The counterweight is then adjusted so the center of gravity of the system is slightly below the knife edge. In this condition, the balanced beam, when set in motion, should have a period of oscillation of at least four to five seconds. Final balancing to the zero point should be done using the primary balance weight. It is neither necessary nor desirable to have the beam at rest in determining the balanced condition.

(d) The test object is placed on the platform such that good contact is attained with the mounting surface and the nose (front) is against the knife edge plate. Care must be exercised to prevent disturbing the adjustment weights on the balance.

(e) The beam is balanced by placing known weights in the weight pan with its knife edge located at a known distance from the knife edge of the balance. Typically, seven inches. Care must be taken to prevent rotational movement of the weight pan along the beam after initial balancing of the beam has been accomplished. If repositioning of the weight pan is necessary, then the procedure must begin from step (a) in all cases. Lines are scribed on the beam at 0.1 inch increments along the beam to be used to determine the distance from the fulcrum (knife edge). The sum of the large weight in the pan can be assumed to occur at the same point. A 1 gram weight is usually moved along the beam for fine balancing. Extreme care must be taken to ensure that the proper distance is measured for this fine adjustment.

(f) The weights should be weighed on the analytical balance prior to use, for maximum accuracy. The weights should be handled with forceps (tweezers) to avoid altering their mass.

(g) The balance is very sensitive to air currents. The most effective and accurate measurements will be made if the final reading is made with the balance enclosed within a confining cover.

(h) When balance is achieved, the distance from the knife edge (fulcrum) to the center of gravity of the test object can be calculated from the moment equation (See Figure 17):

$$d_{cg} = \left[ \Sigma W D_p + w D_f \right] / W_t \qquad (12)$$

where

$d_{cg}$ = distance from nose (front) to center of gravity of test object

$W$ = major weight placed on the balance pan

$w$ = amount of weight used for fine balance

$D_p$ = distance from course weight pan knife edge to fulcrum point

$D_f$ = distance from effective center of fine weight to fulcrum point

$W_t$ = weight of object being measured

(i) The center of gravity along any axis of the test object can be determined by proper orientation.

(j) The accuracy obtained with the center of gravity balance is largely determined by the care used in setting up the balance, the placement of the test device, and the final balancing. Errors can be determined by making measurements from each end of the axis under question and comparing the total with the total measured length of the device. Calibration of the center of gravity can be accomplished using one of the previously discussed calibration models or objects.

b. Center of Gravity Balance Errors

Several errors can be introduced when using the center

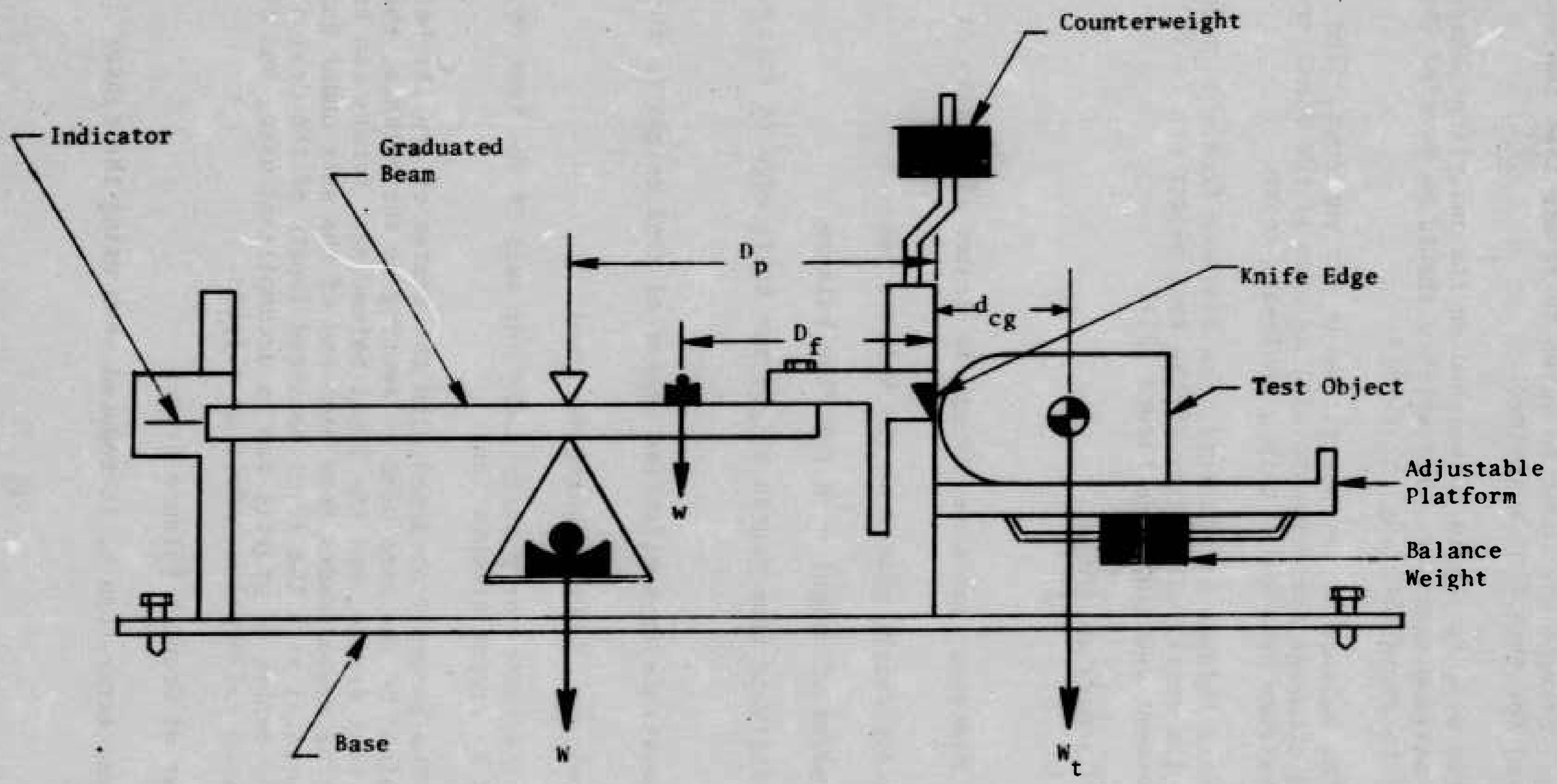



Figure 17. Center of Gravity Schematic

of gravity balance. This may include an erroneous indication of balance due to external forces such as air currents. This can be usually eliminated by use of a cover or case. Another error is caused by improper values assigned to the weights used. As mentioned previously, the weights should be weighed on the analytical scale before use and should be handled only with the proper tool. Tabulation errors may be eliminated by weighing the group of weights each time before they are placed upon the pan.

Poor repeatability can be due to imperfections of the knife edges and knife planes. Extreme care in handling the beam must be taken to avoid damage to these surfaces. When possible, the knife edges should be left in the unloaded position.

An error can be introduced in the balancing operation when small angular errors are allowed from the true level position in both the empty and loaded conditions. The magnitude of this error is dependent on the degree of stability of the movable system and vertical positioning of the test object.

The center of gravity as measured from one end of an object plus the center of gravity measured from the opposite end of the axis should equal the total length of the test object. The difference (error) must be within the allowable test error, which has been determined previously according to the accuracy requirements.

To summarize, two conditions of equilibrium must be satisfied for a body to be in equilibrium. First, there is to be no unbalanced external force on the body; this also implies that the sum of all the forces along any direction is equal to zero. Second, there is to be no unbalanced moments (or torques) on the body. This second condition equilibrium is known as the principle of moments. A moment of force or a torque is the product of the force and its lever arm. The lever arm is measured from the point of rotation along a line that is perpendicular to the line of action of the force. The second condition of equilibrium implies that the sum of the clockwise moments (or torques) about any point is equal to the sum of the counterclockwise moments about the same point.

The center of gravity of a body or object is the point at which a single upward force can balance the weights of all parts of the body for any position of the body. It is also the point about which the algebraic sum of all the torques, due to the weights of all parts of the body, is equal to zero.

## 4. MOMENT OF INERTIA

Inertia, when used to mean the resistance to change in motion, is proportional to mass, and the inertial force or potential per unit of mass varies directly with the rate of change of velocity, $F = ma$ or $m = F(dV/dt)$. Based on this knowledge, moment of inertia has been measured by hanging an object from a wire, twisting it to start it oscillating, then timing the period of oscillation. This method is commonly referred to as the torsional pendulum method and will be discussed in some detail as the basic method of measuring or determining the moment of inertia.

Moment of inertia depends not only on the mass of the object, but also on the distribution of mass with respect to the axis of rotation. Since any number of axes of rotation are possible for an object, there is no singular moment of inertia of an object. This statement is based on the fact that within any object there are three principal axes through any one point: two axes at right angles about which the moments of inertia are a maximum and minimum and the axis at right angles to the plane of these two. The corresponding moments of inertia of the object about these axes are the principal moments of inertia of the object at that point. If that point is the center of gravity of the body, the properties are called the central principal axes and the products of inertia are zero. It is these moments of inertia about the center of gravity that are determined for many technical and scientific operations.

In order to analyze the motion of a rotating object, attention must be focused on a small portion of the object which is at a distance r from the center of gravity point. This is indicated as the shaded area in Figure 18 and has a mass m and a tangential acceleration $a_t$ (Reference 2). Definitions of the basic physical laws concerning rotating bodies are contained in most elementary physics texts.

Many forces may be acting on the shaded mass, but they can be resolved to give one resultant force. This resultant force may then be resolved into the two perpendicular components: centripetal force $[F_c]$ and tangential force $F_t$. From Newton's second law of motion:

$$F_t = ma_t = m\alpha r. \tag{13}$$

Where $\varphi$ (as in Equation (13)) is the angular acceleration. The moment of the resultant force about the center of gravity is the tangential force acting at the distance r. This is further known as the resultant torque acting upon the mass m. If the previous equation is multiplied through by r, the equation for the resultant torque is:

$$rF_t = \left[mr^2\right]\alpha. \tag{14}$$

If the symbol M is used to indicate torque, then the equation is written:

$$M = \left[mr^2\right]\alpha. \tag{15}$$

Reference:
2. Alexander, J.; Pomeranz, K.; Prince, J.; Sacher, D.: Physics for Engineering Technology, John Wiley & Sons Inc., 1966.

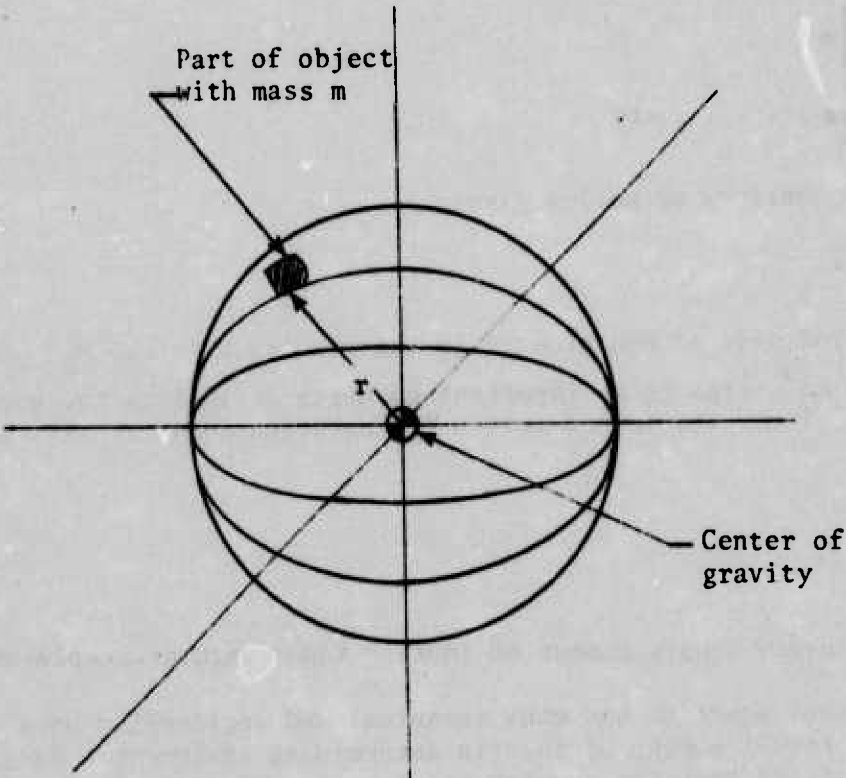



Figure 18. Moment of Inertia for Rotating Object

This is the equation of motion for the small shaded portion m of the rotating object. Similarly, equations can be written for all the other portions of the rotating object. These other portions may have a different mass m and a different radius r, but all portions of the rotating object have the same angular acceleration $\alpha$. The equations of motion for these various parts are:

$$M_1 = \left[m_1 r_1^2\right]\alpha$$

$$M_2 = \left[m_2 r_2^2\right]\alpha$$

$$M_3 = \left[m_3 r_3^2\right]\alpha \text{ , . . . . , etc} \qquad (16)$$

Summing these equations of motion gives:

$$\Sigma M_i = \left[\Sigma m_i r_i^2\right]\alpha \qquad (17)$$

The left hand side of Equation 17 is the resultant torque $M_R$. The sum of all the $mr^2$ terms is an important quantity defined as the moment of inertia, I. Thus, the law of motion for rotation about an axis may be written as

$$M_R = I\alpha \qquad (18)$$

or:

Resultant torque equals moment of inertia times angular acceleration

This important equation has many technical and engineering uses and is the basis for most moment of inertia determining equipment. Basically, this equation states that the angular acceleration of a rotating object is directly proportional to the resultant torque acting on the object.

The moment of inertia I is a measure of the rotational inertia of a rotating object. An object with a large moment of inertia possesses a great deal of rotational inertia and would require a large torque to produce a comparatively small angular acceleration. On the other hand, objects with a small moment of inertia possess only a small amount of rotational inertia, and these objects may be given a large angular acceleration with a comparatively small torque. In some simple cases, such as for the objects used for calibration purposes, it is possible to calculate the moment of inertia directly from the sum of all the $mr^2$ terms.

$$I = \Sigma m_i r_i^2 \qquad (19)$$

This summing procedure is possible only if the object consists of a number of separate masses, but this usually is not the case. For rotating objects that consist of a continuous distribution of matter, the sum must be evaluated using integral calculus. This has been done for many simple figures and/or objects, and the results of the calculations may be found in the engineering handbooks. The moments of inertia for some common objects used for calibration purposes are indicated in Figure 19. Note that Figures 19(b) and 19(c) are for the same type objects but the axis of rotation is different, and it is the variation of axis that determines the proper formula or equation.

When a mass M is suspended from a wire twisted as in Figure 20 and released, the periodic oscillations that take place are simple harmonics. The principle shown is referred to as a basic torsion pendulum. The wire is basically a spring. When a spring is stretched within its elastic limit, the spring or wire exerts a restoring force (torque) which is opposite in direction to the elastic deformation. When the wire or spring is compressed, it will again exert a restoring force opposite in direction to the elastic deformation. If this deformation is indicated by the angle of twist $\theta$, then an equation for the restoring torque M can be written as

$$M = -k\sin\theta \tag{20}$$

The proportionality constant k depends on the composition of the wire, the minus sign indicates that the elastic deformation and the restoring force are always in opposite directions, and sin $\theta$ indicates the displacement of the wire with respect to its equilibrium or unstressed position. If the elastic limit is not exceeded, the same potential energy exists when the wire is displaced in either direction by sin $\theta$ units of deformation.

From the previous discussion, it was determined that torque is also written as

$$M = I\alpha \tag{21}$$

Where:

I = moment of inertia of a mass m about an axis through its center of gravity.

$\alpha$ = angular acceleration

If Equation (21) is combined or equated to Equation (20), then

$$M = I\alpha = -k\sin\theta \tag{22}$$

such that

$$\alpha = \frac{-k}{I}\sin\theta \tag{23}$$

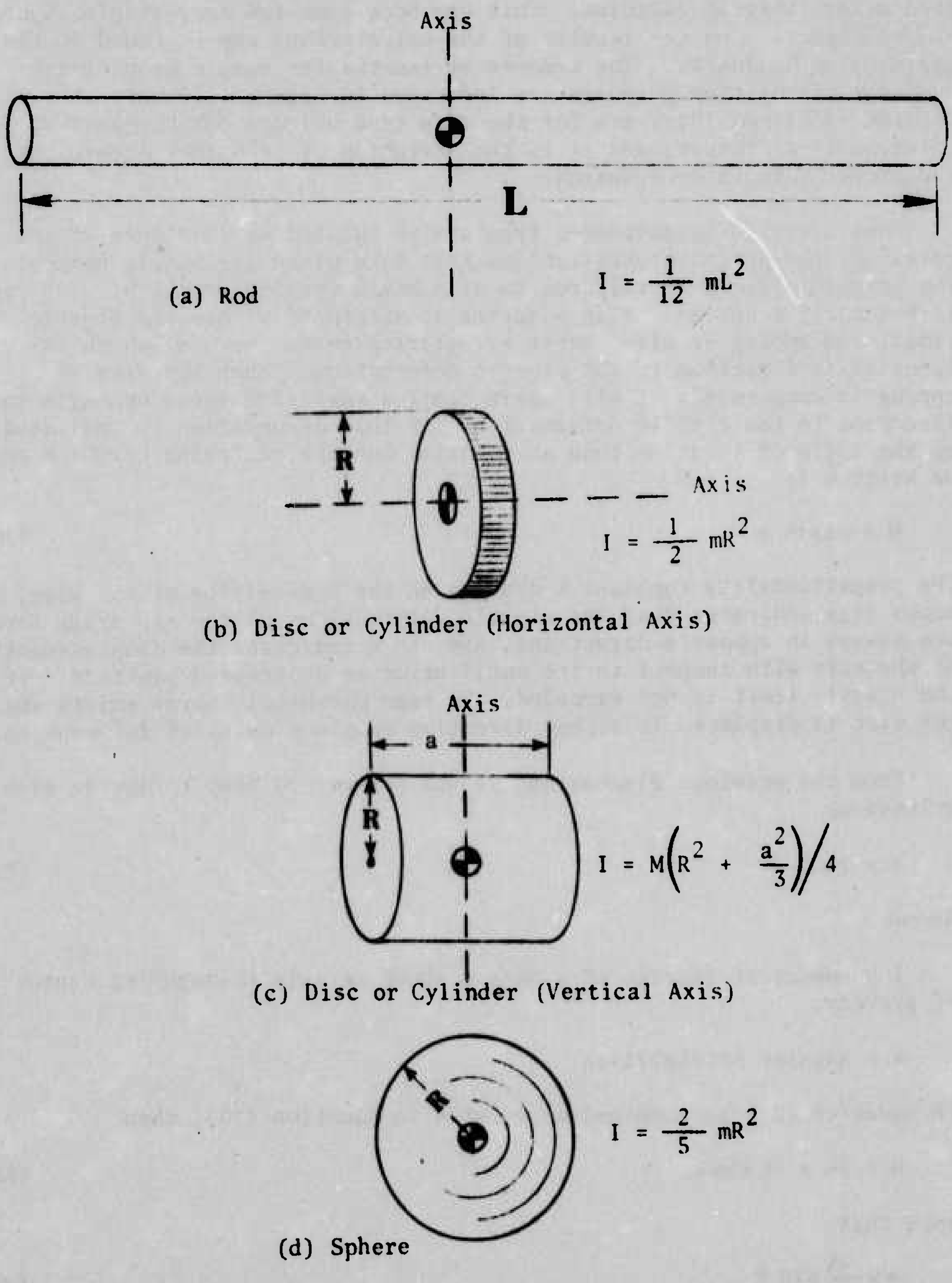



Figure 19. Moment of Inertia of Calibration Objects

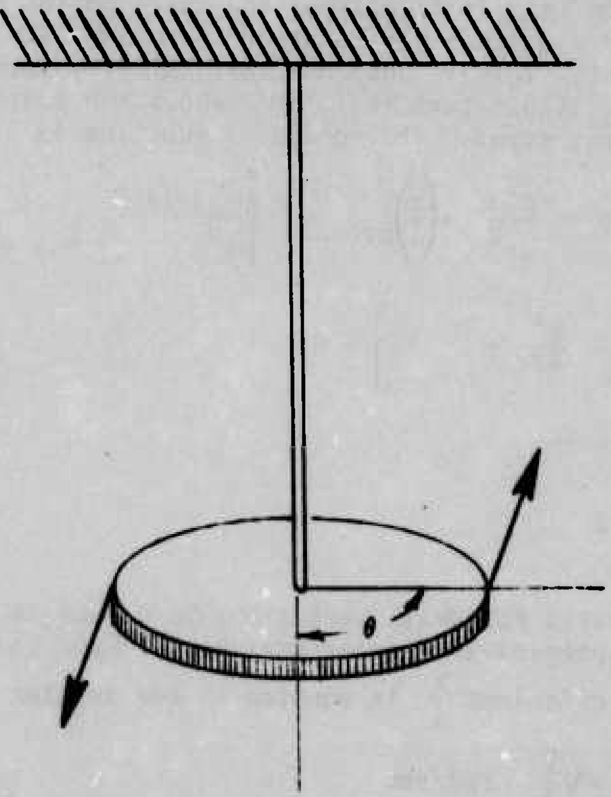



Figure 20. Torsion Pendulum

If consideration is made totally of the small motion about this reference point, i.e., $\theta = 0$, then the equation of motion can be linearized about this operating point. This is done mathematically by forming a Taylor series expansion of the nonlinear term $(k/I)\sin\theta$ about the point $\theta = 0$ and retaining only the first-degree terms. The nonlinear equation is

$$\frac{d^2\theta}{dt^2} + \frac{k}{I}\sin\theta = \frac{d^2\theta}{dt^2} + \left(\frac{k}{I}\right)\sum_{n=o}^{\infty}\frac{\theta^n}{n!}\left[\frac{d^n(\sin\theta)}{d\theta^n}\bigg|_{\theta = 0}\right] =$$

$$\frac{d^2\theta}{dt^2} + \left(\frac{k}{I}\right)\left[\theta - \frac{\theta^3}{3!} + \ldots\right] = 0 \qquad (24)$$

the linear equation is

$$\frac{d^2\theta}{dt^2} + \frac{k}{I}\theta = 0 \qquad (25)$$

This equation is valid for small variations in $\theta$, and it describes simple harmonic motion expressed in angular variables. From this equation, the linear constant coefficient $\frac{k}{I}$ is equated to the angular velocity

$$\omega^2 = \frac{k}{I} \quad \text{or} \quad \omega = \sqrt{\frac{k}{I}} \quad \text{rad/sec} \qquad (26)$$

Some useful relationships from simple harmonic motion exist. One states that frequency of rotation f is inversely proportional to the time for one period, i.e., $f = \frac{1}{T}$. Another relationship states that angular velocity is directly proportional to frequency multiplied by two radians, i.e., $\omega = 2\pi f$.

After the necessary substitutions are made, the following equations are derived:

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{k}} \qquad (27)$$

Where:

$T$ = the time for one period

$f$ = the frequency of rotation

Equation (27) can then be transposed to provide the essential simple equation

$$I = kT^2 \qquad (28)$$

Where k is determined as the spring constant of the torsion wire.

When the torsion pendulum method is used, frequently the object swings from side to side and bounces up and down rather than rotating smoothly about an axis. These motions make accurate data acquisition difficult. Furthermore, several practical problems are involved in hanging most devices upside down: attaching the wire, hanging the entire system, calibrating the device, and correcting for change in calibration when the weight of the test object stretches the wire. For these reasons, the moment of inertia is frequently calculated instead of measured, but this is not practical when the object is of an odd shape or of a composite material. After considerable calculation, the accuracy of the moment of inertia value may still be in doubt.

The magnitude of the task of measuring moment of inertia has been reduced by a bench top instrument available in the Projectile Measurements and Instrumentation Laboratory. This unit (Reference 3) operates on the principle of the inverted torsional pendulum. Instead of hanging from a torsion rod or wire, the test object is retained in a fixture attached to the top of a vertical torsion member. Precision bearings constrain the motion of this torsion member to pure rotation. A sensing device determines the period of oscillation of the torsion member adapter and test object system.

The commercially available instruments shown in Figure 21 can determine in a matter of seconds the period of oscillation of an object under test and display the parameter on an interfacing digital display counter (center). The time read from the counter is applied to a simple formula ($I = kT^2$) so that the answer is calculated directly; no geometric juggling, parallel axis theorems, or extensive mathematical exercises are required.

The moment of inertia instrument contains a nickel-alloy torsion fiber that eliminates horizontal and vertical motion to produce one-degree-of-freedom operation. The fiber has been preloaded in tension to approximately 10 times the maximum permissible weight of a test object. This eliminates variations in spring constant or rotational center as a function of test object weight. This fiber is clamped at its midpoint to a hollow oscillating tube that runs through the center of the instrument. The ends of the tube are restrained by cross-axis flexures (0.5% accuracy). To keep tare inertia as low as possible, the oscillating tube is fabricated of lightweight aluminum or magnesium.

Hysteresis damping of the flexure member in all torsional pendulums causes a decay of oscillation amplitude which affects measurement of the period of oscillation. This source of error is minimized by using a nickel-

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Reference:

3. Operations Manual for Inertia Dynamics Moment of Inertia Instrument. Technical Bulletin 722, Inertia Dynamics, Incorporated.

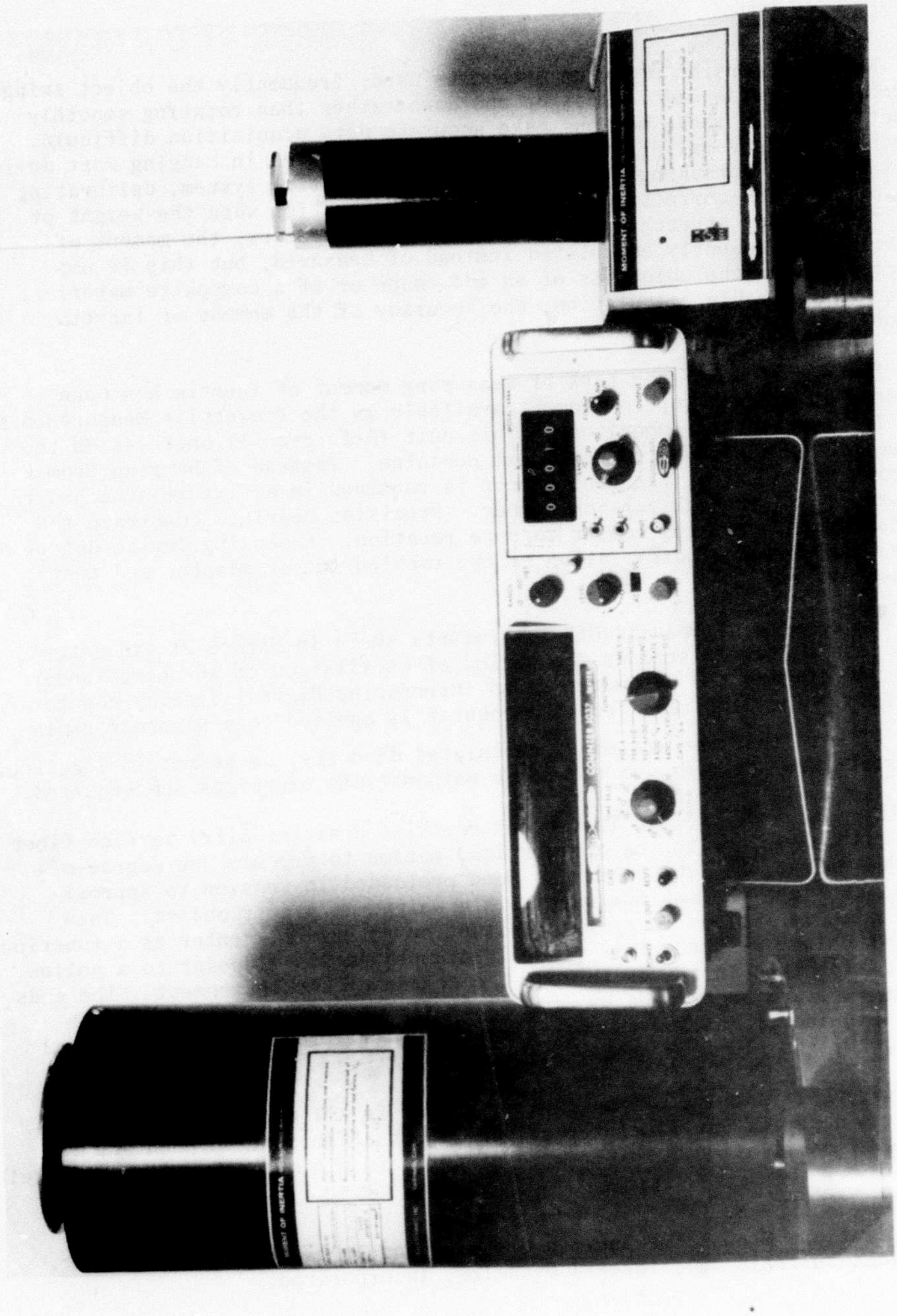



Figure 21. Moment of Inertia Instruments

alloy flexure member with extremely low hysteresis damping. Additional restrictions allow all oscillations to be initiated with the same angular displacement, and this results in a more accurate period timing.

The fringe pattern and incident light problems associated with conventional photo-electric timing methods are solved by using a magnetic sensing device. This device produces a short pulse (microsecond width) just as the oscillating tube passes through its center of oscillation. These narrow pulses allow the instrument to interface with a variety of electronic counters using either AC or DC coupled input sections, and this procedure permits accurate timing over a wide range of input threshold settings.

A gallium arsenide light-emitting diode mounted on the front panel of the instrument monitors the output of the trigger circuit. The first pulse starts a conventional electronic period counter. When the present plug-in unit is used, the counter is stopped when a specified number of counts is reached. Typically, a power of 10 is used in order to make direct average reading from the counter easier.

Calibration with a special test weight of known moment of inertia establishes the value of k for a particular instrument. The calibration object is an accurately measured cylinder with a threaded concentric hole in its center. A threaded rod of the same height as the threaded hole screwed almost all the way in creates a mounting stud. Thus, the moment of inertia of the test weight is that of a simple cylinder, i.e.,

$$I_R = 0.5Mr^2 \text{ (gm-cm}^2\text{)} \tag{29}$$

Where:

M = mass in grams

r = radius in centimeters

Time period measurements with and without this calibration object permit calculation of the calibration constant k in gm-cm; the moment of inertia computed with this k has the units gm-cm$^2$.

Using the equation $I = kT^2$ (the counter indicates T) the tare inertia ($I_t$) for the instrument and the system inertia ($I_s$) for the instrument/specimen combination can be determined. One subtraction operation then yields the moment of inertia of the test object ($I_o$).

$$I_o = I_s - I_t \tag{30}$$

Conversion of the moment of inertia from oz-in-sec$^2$ to another system of units can be accomplished using the chart in Appendix A.

There are four basic limitations on the size of objects which can be measured on the two instruments available in the Projectile Measurements and Instrumentation Laboratory:

(a) Weight. The maximum allowable total weight of object and test fixture is 5 pounds for one instrument and 40 pounds for the second. Exceeding these limits may damage the instrument.

(b) Moment of Inertia. There is no limit on the maximum moment of inertia which can be measured. The minimum moment of inertia is limited by the random timing error of the instrument. Each instrument has a particular tare moment of inertia which must be subtracted from the total measured inertia. When the test object becomes small relative to the tare of the instrument itself, then the measured value of the test object will result from a small difference between two measured time periods. The relatively small random fluctuation in the measured time periods now becomes significant in terms of the measured value of the test object. The manufacturer has determined that the random timing error for the 0.5% instrument is about 0.001 times the tare moment of inertia. In order to maintain the 0.5% maximum error, the minimum moment of inertia must be limited to 200 times 0.001, or about 1/5 of the tare.

(c) Height. The test fixtures supplied with the machine and those that have been specially machined for ballistic models have been ground internally and on their interface; the result is a very small angle error that is normally insignificant. However, long thin objects, when standing upright, will lean slightly to one side due to this finite error. Since these long thin parts also have a small radius of gyration, there is a limit on the ratio of the length of the part to its radius of gyration. The manufacturer has determined that the distance between the test table and the center of gravity of the test object must be less than 36 times the radius of gyration to remain within the stated accuracy limits of the instrument. For most cylindrical objects, this ratio corresponds to a height-to-diameter ratio of 24:1.

(d) Center of Gravity offset. An offset condition may exist when the center of gravity of a test object does not coincide with its rotational axis. If the center of gravity is offset, then a special test fixture may have to be used to avoid introducing errors. Objects with a product of center of gravity offset times weight that is less than 250 kg-cm can be measured directly. No balancing or locating of the center of gravity is required. The resulting measured value will be about the rotational axis of the object and not the principal axis or center of gravity of the test object. Generally, this is the value of interest; however, if the moment of inertia about the principal axis (CG) is desired, then it may be calculated from the measured value, the mass of the test part, and the center of gravity location.

Methods for direct offset center of gravity measurement and for heavier objects with large center of gravity offsets may be found in the operation manual provided with the instruments.

a. Moment of Inertia Measurement Procedures

Before accurate moment of inertia measurements can be obtained, the following must be accomplished:

(a) The instrument is leveled using the bubble level and the adjusting screws.

(b) The starting lever is moved slowly to the start position and held there. Then the lever is released sharply; the torsion pendulum oscillates, and the indicator light flashes in synchronism with this oscillation.

(c) A digital period counter is connected to the instrument, and its controls are adjusted so that the oscillation period of the object may be measured. The instrument is started several times. The period indicator should consistently read about 70 to 100 miiliseconds (varies with the model). Variations between readings should be less than 0.1%.

(1) Setting up the Counter

The timing circuit of the moment of inertia instruments produces a narrow pulse that is the optimum signal for any period counter to measure. For this reason, most counters will operate over a wide range of input settings.

The counter controls are set as follows:

| | |
|---|---|
| Power | On |
| Time Base | Set for desired time period ($10^{-7}$ typical) |
| Display | Hold (clockwise to switch off position) |
| Function | Per A |
| Range | 1 |
| Level | Zero |
| AC/DC | AC |

(The following controls are on the preset plug-in)

| | |
|---|---|
| Slope | + (positive) |
| AC/DC | DC |
| Range | 1 |
| --N-- | 10 (for an average of 10 periods: Remember to divide time by ten) |
| Trigger Level | Off zero to positive side |
| Function Switch | NxP |

(2) Calibration

Using one of the calibration objects available, the calibration constant k is calculated as follows:

$$k = \frac{I_c}{T_c^2 - T_o^2} \tag{31}$$

Where:

$I_c$ = Moment of inertia of test weight in gm-cm$^2$.

$T_c$ = Period of oscillation in seconds with calibration weight mounted on the instrument.

$T_o$ = Tare period of oscillation of instrument and any holding fixture with calibration weight removed.

To obtain the most accurate results, the calibration inertia ($I_c$) should be approximately that of the unknown test object.

The time period measurements should be repeated several times. The time period should not vary more than 0.1% with the same payload. False readings may occur occasionally due to electrical noise or to the operator accidentally touching the test object.

The instrument should be recalibrated at intervals of three months or less. To be certain of achieving specified accuracy, the instrument should be calibrated just before making a measurement.

Calibration is valid for use only with the same digital period counter. Whenever the digital counter is changed or recalibrated the unit <u>must</u> be recalibrated.

(3) Test Fixtures

Test fixtures are available to hold objects with standard shaft and hole sizes. If the object to be measured does not fit any of these fixtures, then a fixture must be manufactured to suit the test object.

The design of test fixtures for the moment of inertia instrument does not require the same care as a test fixture for balancing or vibration testing. Dimensional accuracy is not critical. The fixture need not be particularly strong or rigid. Criteria to follow are:

(a) The fixture should have as small a moment of inertia as possible. To achieve this, it will be made out of aluminum, keeping the diameter to a minimum and using lightening holes as practicable.

(b) In order to simplify measurement of the fixture, tare mounting hardware should be avoided. All mounting hardware must be included in tare reading.

(c) The fixture must have a positive method of positioning the axis of the test object.

(d) The fixture mounting hole should be threaded UNC 10-24 2B to a depth of 3/8 inch, counterbored 0.250/0.251 diameter by 3/16 inch deep, if used with the smaller instruments. The large instruments require UNF 1/4-28 to a depth of 1/2 inch, counterbored 0.376/0.377 diameter by 7/32 inch deep.

(e) Fixtures for tall, thin objects should have an angle error of less than 10 minutes (1/6 degree).

(4) Operating Procedure

(a) The weight of the test object is measured. It should be within the capacity of the particular instrument used. If it exceeds the instruments capacity, it may be possible to disassemble the test object, measure the moment of inertia of the individual parts, and then sum the measurements for the total moment of inertia.

(b) A test fixture is selected or fabricated to mate with the object being measured.

(c) The empty test fixture is mounted on the instrument, and the period of oscillation $T_o$ is measured in seconds. The total tare moment of inertia, $I_o$, of the instrument plus mounting fixture is now calculated:

$$I_o = kT_o^2 \tag{32}$$

Where k is the calibration constant for the instrument.

This tare moment of inertia is subtracted from all measurements made using this particular fixture. Equation 32 should be resolved to at least five significant figures; otherwise, considerable error results when the test object moment of inertia is small relative to the tare moment of inertia.

(d) The object to be measured is mounted in the test fixture and a new period of oscillation $T_*$ is obtained. The total moment of inertia can now be calculated:

$$I_* = kT_*^2 \tag{33}$$

(e) The resulting value of $I_*$ is the combined moment of inertia of the test object and the tare. The moment of inertia of the test object alone is found simply by subtracting the tare as follows:

$$I \text{ (of test object)} = I_* - I_o \tag{34}$$

(f) The value of the moment of inertia will be expressed in units of gm-cm$^2$ (since these units were arbitrarily chosen when defining the value of the calibration constant k). Conversion to another system of units may be accomplished by using the chart provided in Appendix A or by referring to an engineering manual which lists conversion factors for mass, distance, and acceleration of gravity. If it is desired to always express moment in another set of units, then the value of k may be divided by the appropriate factor given in the chart.

These procedures are summarized in the following example:

The moment of inertia of an object is to be found in the units of oz-in-sec$^2$. The calibration constant for the instrument is already known and is 0.100 oz-in-sec$^2$.

(a) The tare moment of inertia (instrument plus collets, test fixtures, etc) is determined. Tare time period is measured and is found to be .08 second.

The formula is applied to obtain tare moment of inertia:

$$I \text{ (tare)} = kT^2 \text{ (tare)} \tag{35}$$

$$I \text{ (tare)} = .100 \times .08^2 = .100 \times .0064 = .00064 \text{ oz-in-sec}^2. \tag{36}$$

(b) The total moment of inertia (test object plus tare moment of inertia) is determined. The test object is mounted and the total time period measured and found to be 2.0 seconds. Applying the formula to obtain the total moment of inertia:

$$I \text{ (total)} = kT^2 \text{ (total)} \tag{37}$$

$$I \text{ (total)} = 0.100 \times 2.0^2 = 0.100 \times 4.0 = 0.4 \text{ oz-in-sec}^2. \tag{38}$$

(c) The moment of inertia of the test object is calculated by subtracting the tare moment of inertia from total moment of inertia.

$$I \text{ (total)} - I \text{ (tare)} = I \text{ (test object)} \tag{39}$$

$$0.4 - 6.00064 = 0.39936 \text{ oz-in-sec}^2 \tag{40}$$

b. Precautions for Error Reduction

Special care should be taken to prevent introduction of error into the measurement. The top of the instrument test table must not be struck or pressure applied in excess of the total weight capacity of the instrument. Often it is desirable to temporarily change the position of the test object. The procedure is to unscrew the fixture, mount the test object in the fixture, and then screw this assembly back on the instrument. This avoids any stress on the torsion fiber when mounting the payload. Extreme care must be used when screwing any fixture to the instrument as the interface threads are soft and can easily be damaged if abused.

When the starting lever is used, oscillation should be initiated by slowly moving the start lever to its full start position and holding it there until the test part has stopped all motion. Failure to do so imparts energy to the oscillation and introduces timing errors.

Test objects to be measured should be rigid and should be tightly fastened to the fixture. The fixture should be tightly screwed to the instrument test table. It is not possible to measure parts containing internal bearings or flexible assemblies without first making the assembly rigid.

All calculations must be done to at least five place accuracy when the payload moment of inertia is small relative to the tare moment of inertia. Each period reading should be repeated at least three times. True readings will usually not vary more than 0.1%. False readings will occur infrequently and will show large random variations. Counter adjustments and mechanical tightness should be checked if readings vary more than 0.5%.

To obtain maximum accuracy, the instrument should be calibrated just before using. Ten readings are taken and the norm value - that value which is displayed most frequently - is obtained. This is preferable to taking the average which is sensitive to the random false values that are sometimes observed. Another good method for improving accuracy is to set the period counter so that it resets itself at frequent intervals and thus can display successive readings as the test object oscillates after being started. This method has the advantage of eliminating the problems inherent in manually starting the oscillation for each individual single period reading.

MEASUREMENTS MUST BE MADE ABOUT THE CENTER OF GRAVITY -- These instruments are designed for use with balanced test objects. The center of gravity position can be measured from the reference end of the test object using the optical comparator. An inscribed mark made on the test object can then easily be lined up with the center of gravity position of the test fixture. Objects with a large center of gravity offset should be measured about their center of gravity and the measured moment of inertia increased by $d^2M$, where d is the distance between the rotational axis of the object and its center of gravity axis, and M is the mass of the test object.

The accuracy of the moment of inertia instrument is limited by the accuracy of the period counters used to measure the time period. Most counters which use a crystal controlled time base are sufficiently accurate so that no significant error results. Counters which use the power line frequency for a time base will introduce significant errors.

A battery is used in these instruments because of its constant voltage and ripple free characteristics. The instruments draw current continually while they are switched on; therefore, it is recommended that the moment of inertia instrument switch be turned off after each test to avoid any significant voltage change between tests.

On the larger instruments, a hardened dowel pin is provided to help in mounting the test object, collets, etc. The pin is inserted into the hole in the side of the instrument test table and is used to take the torque reaction when screwing a fixture to the instrument. Torque on the instrument starting lever in excess of light finger pressure should be avoided.

SECTION III

EQUIPMENT SPECIFICATIONS

This section provides specifications of the equipment utilized in the Projectile Measurement and Instrumentation Laboratory. A physical description of each piece of specialized equipment is included.

The selection of an instrument for a specific mass property measurement requires consideration of several possible types and comparison of many factors. The range of the instrument must equal the expected range of the variable to be measured. Excessive instrument range means a loss of readability. For a wide range in an associated measurement, multiple-range instruments are desirable.

The sensitivity and readability of the instrument require careful consideration. The influence which small variations in each physical quantity have on the final result of the measurement determines the minimum sensitivity of each instrument in relation to the required accuracy of the result. A sensitivity that is too great involves unnecessary expenditure of time and attention in procuring the data, while insufficient sensitivity makes the entire measurement worthless for the intended purpose. This is to say that it is impractical to measure large objects with a tape marked off in microns or small objects with a ruler indicating only centimeters. Common sense must be used such that the instrument meets the needs of the measurement.

The following specifications are presented in order that proper selection can be made easier.

1. OPTICAL COMPARATOR AND MEASURING MACHINE

An optical comparator is many pieces of equipment combined into one. It is the most versatile measuring instrument available today. The optical comparator provides for enlargement of test objects for inspection and measurement. All lenses will magnify a part exactly the rated number of times. For example, a 1 inch diameter pin will measure exactly 10 inches on the comparator screen using a 10 magnification lens system.

The optical comparator (Figure 22) used in the Projectile Measurements and Instrumentation Laboratory combines operator-oriented side screen design with extremely close capability for precision measurements. The screen is located away from the projection and reflection lens axis, thus eliminating any distracting reflections of the illumination sources. An operator can comfortably stand as close to the screen as he wishes and at the same time easily manipulate table controls for precision measurement. The screen is tilted 5 degrees for ease of viewing and for holding overlay charts firmly in place. Overlay charts can be installed in moments.

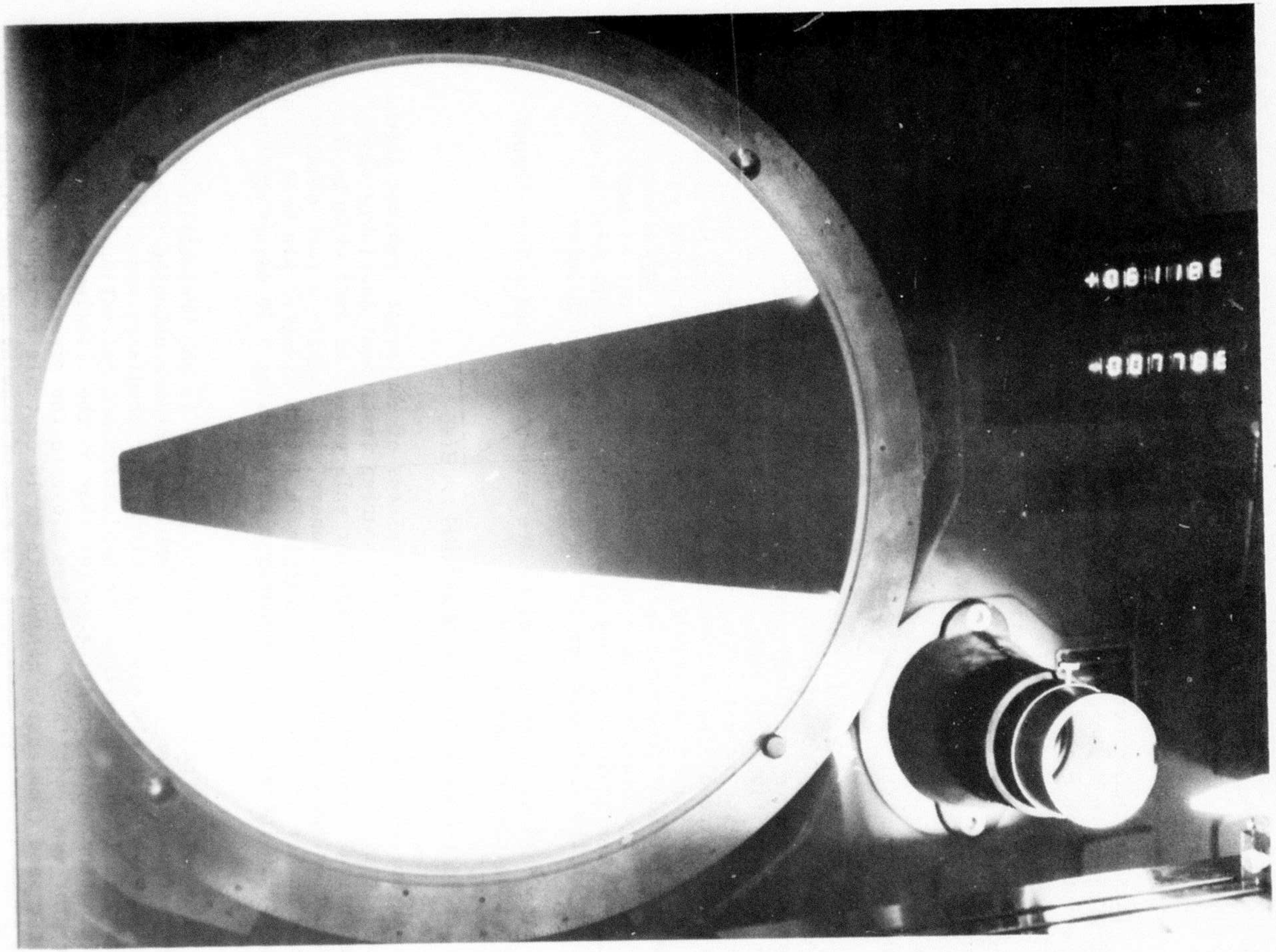


Figure 22. Shadow Outline from Optical Comparator

The optical comparator can be equipped to perform all types of optical inspection work to a high degree of accuracy. The features include telecentric and erect image optical systems, mercury arc illumination, automatic lens selector, automatic table position indicator, power elevation, direct reading measuring facilities, extremely large capacity, reflection, tracing and photographic attachments, and easily adjustable chart rails. Horizontal and vertical measurements are made through electronic power with lead screw control. Automatic coordinate motions can be made either independently or simultaneously. Each motion can be automatically zeroed at any point within the table travel for fast accurate measurement at any time. All measurements are read as actual dimensions, without computation from a digital display, in increments of 0.0001 inch or 0.001 mm.

Controls for both horizontal and vertical table travel are conveniently located. Variable speeds for fingertip control of fine measurements and high speeds for long traverse are available through a single accelerator for each table motion. Instant zeroing for digital display is automatic by a simple push button.

The precision work table is supported on two heavy duty columns. This rugged, double spindle construction permits accurate motion and measurement of work pieces weighing up to 200 pounds. The table top has two fixture slots, and it is mounted on roller bearings for fast, accurate traverses. It is possible to project, over and under, objects up to 8 inches in diameter by 20.75 inches long.

Angles from 0° to 360° can be read accurately and rapidly to one minute of arc. The chart ring is rotated to match the shadow outline (Figure 22) with either the vertical or horizontal chart center line. Angle measurement is then read directly, to one minute of arc, on the vernier. The direct reading vernier and graduated chart ring do away with any chance of reading error and eliminates all time consuming calculations.

The table angle adjustment up to 15°, clockwise and counterclockwise, provides a direct method for compounding the table to position the helix of threads parallel to the optical axis of the machine. The slide used for this table motion may be removed to increase the lens height above the table by 2 inches.

Mercury arc illumination provides a steady, non-flickering symmetrical point source of light. Screen illumination is 5 to 10 times greater than that provided by any filament type of light source.

All images are erect and maintain a fixed focal plane. Focal clearance of 13 inches is constant at all magnifications. The standard projection lenses are 10, 20, and 31.25 magnifications, and other lenses can be added later to provide 50, 62.5, 100, and 200 magnification.

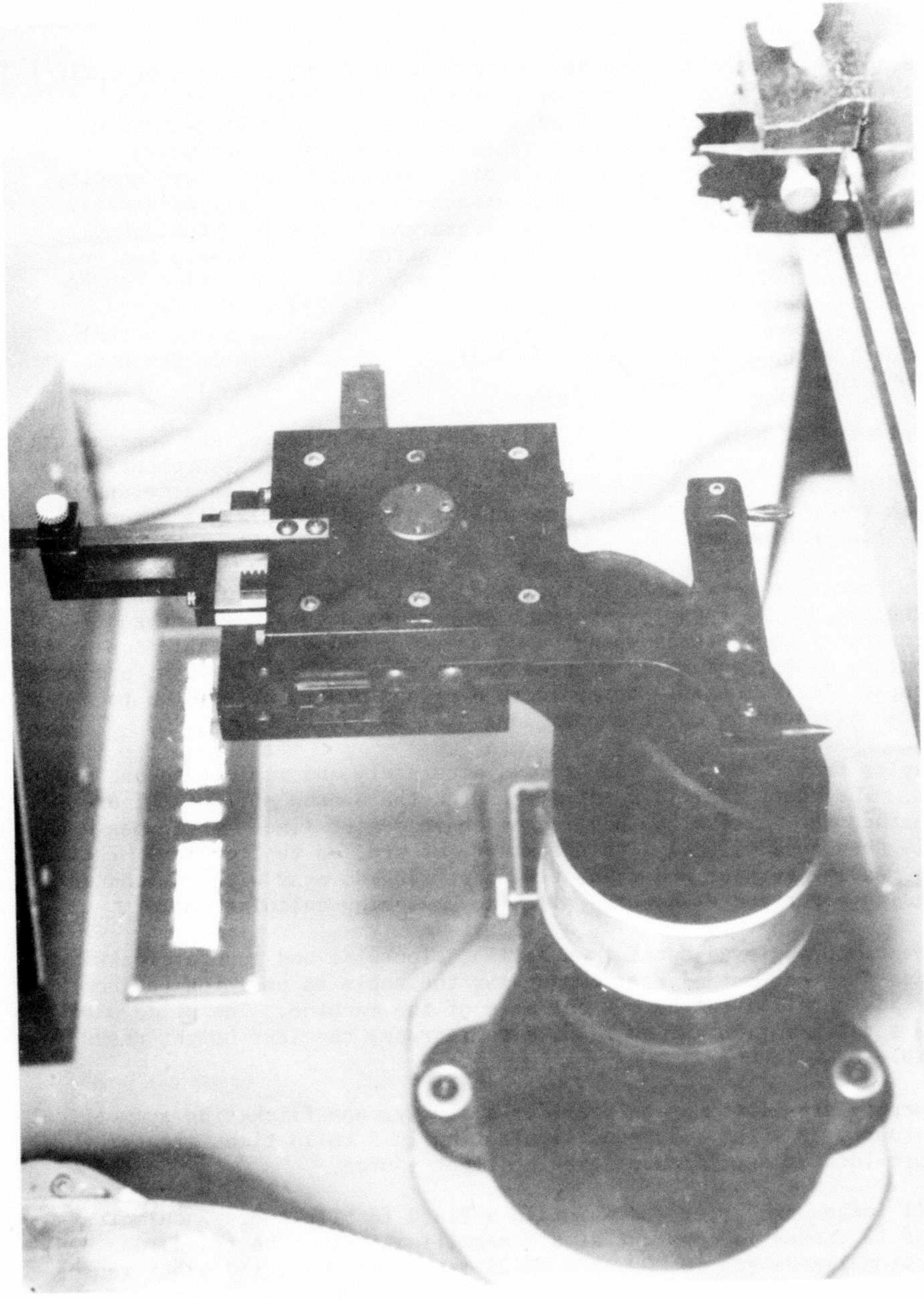


Figure 23. Tracing Unit for Optical Comparator

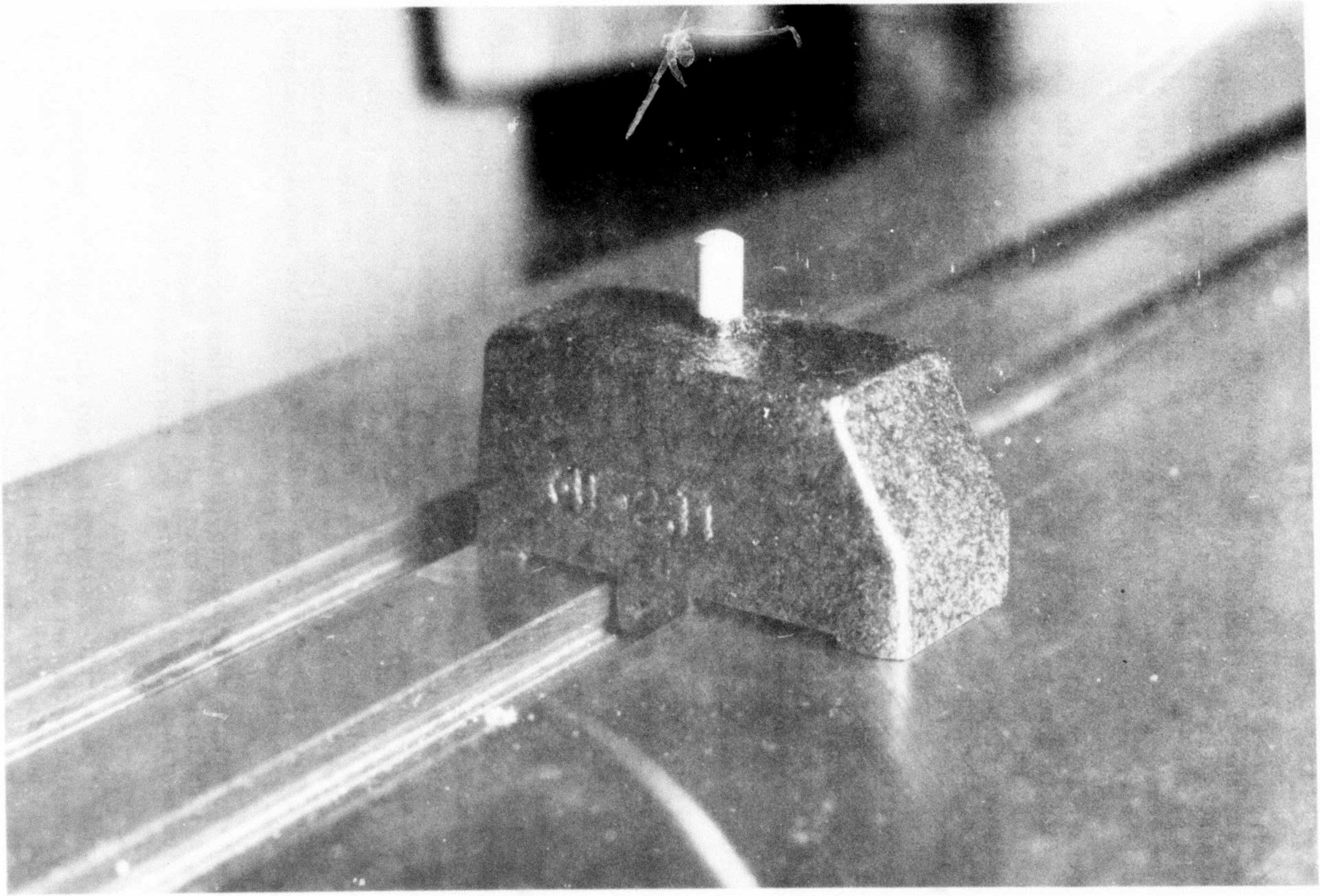


Figure 24. Chart Alignment Fixture for Optical Comparator

A tracing unit, Figure 23, is equipped with two styluses mounted on a tracing arm that is attached to a 3 by 3 inch coordinate slide. One stylus contacts the object surface. Its movement over the contour is duplicated by the second stylus which lies within the focal plane of the projection lens. The enlarged image of the projected stylus follows a chart outline as it appears on the viewing screen.

The chart alignment fixture in Figure 24 is used to align the index screen. This is done by setting the graduated chart ring to zero and then rotating the chart within the chart ring until the projected shadow of the aligning fixtures radius pin remains tangent to the horizontal centerline of the chart when it is moved laterally in front of the projection lens.

The versatility of the comparator is extended through the use of standard holding fixtures (Figures 25 and 26). Various precision holding stages not necessarily shown include matched pairs of V-blocks, matched pairs of center and center block assemblies, vise stages, rotary tables, and vertical or constant centerline V-block assemblies.

## 2. WEIGHING APPARATUS

Various scales or balances are provided for weighing test objects. The various units provide capabilities for weighing objects up to 45 pounds.

The top loading balance, shown in Figure 8, has a weighing range of 1000 grams and a readability of 0.01 grams. This item has a special release feature. In order to release the transport arrestment, the black stopper shown at the bottom of Figure 27 is removed. Then the white knob above this hole is removed and inserted into the lower hole. After turning to release the balance, the white knob is replaced in the upper hole, and the black stopper is reinserted into the lower hole. A second top loading balance (Figure 9) has a weighing range of 3000 grams and a readability of 0.1 gram. Either of these top loading balances give weighing results almost as fast as the test object can be placed on them. The readout in each case is in large legible numerals designed for fast, easy reading of weighing results.

The metric triple beam balance (Figure 6) has a capacity of 2610 grams and a sensitivity of 0.1 gram. It features magnetic damping for speed, agate bearings for long life, and tiered beams for complete visibility.

The combination metric-avoirdupois heavy duty solution balance (Figure 7) completes the set of balances. This balance has the capacity of 20 kg or 45 pounds, with a readability or sensitivity of 1 gram or 1/16 oz. This ruggedly constructed balance is ideal for weighing bulk solutions or large items. It features an end reading device which permits

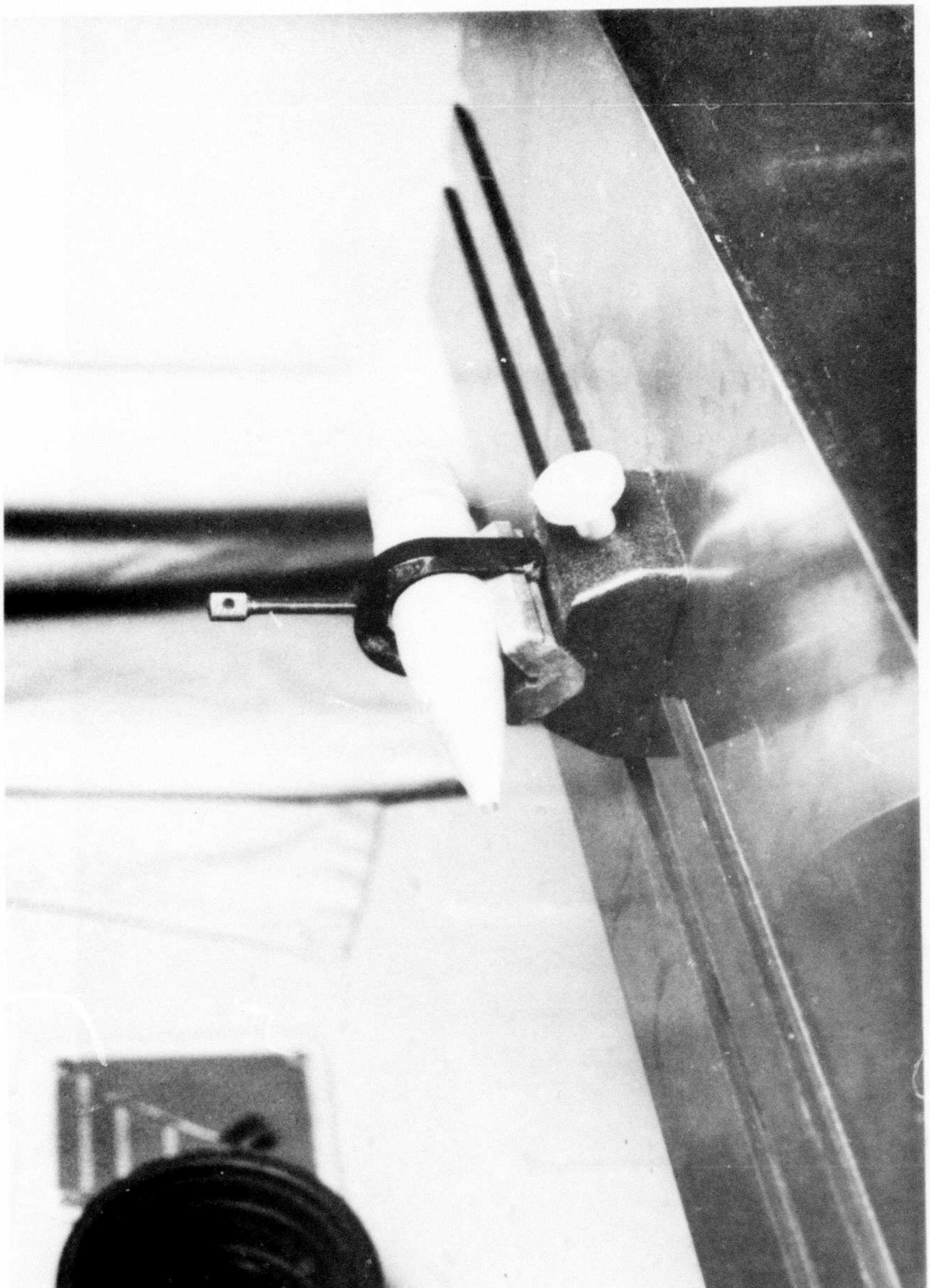


Figure 25. V-Block Holding Fixture for Optical Comparator

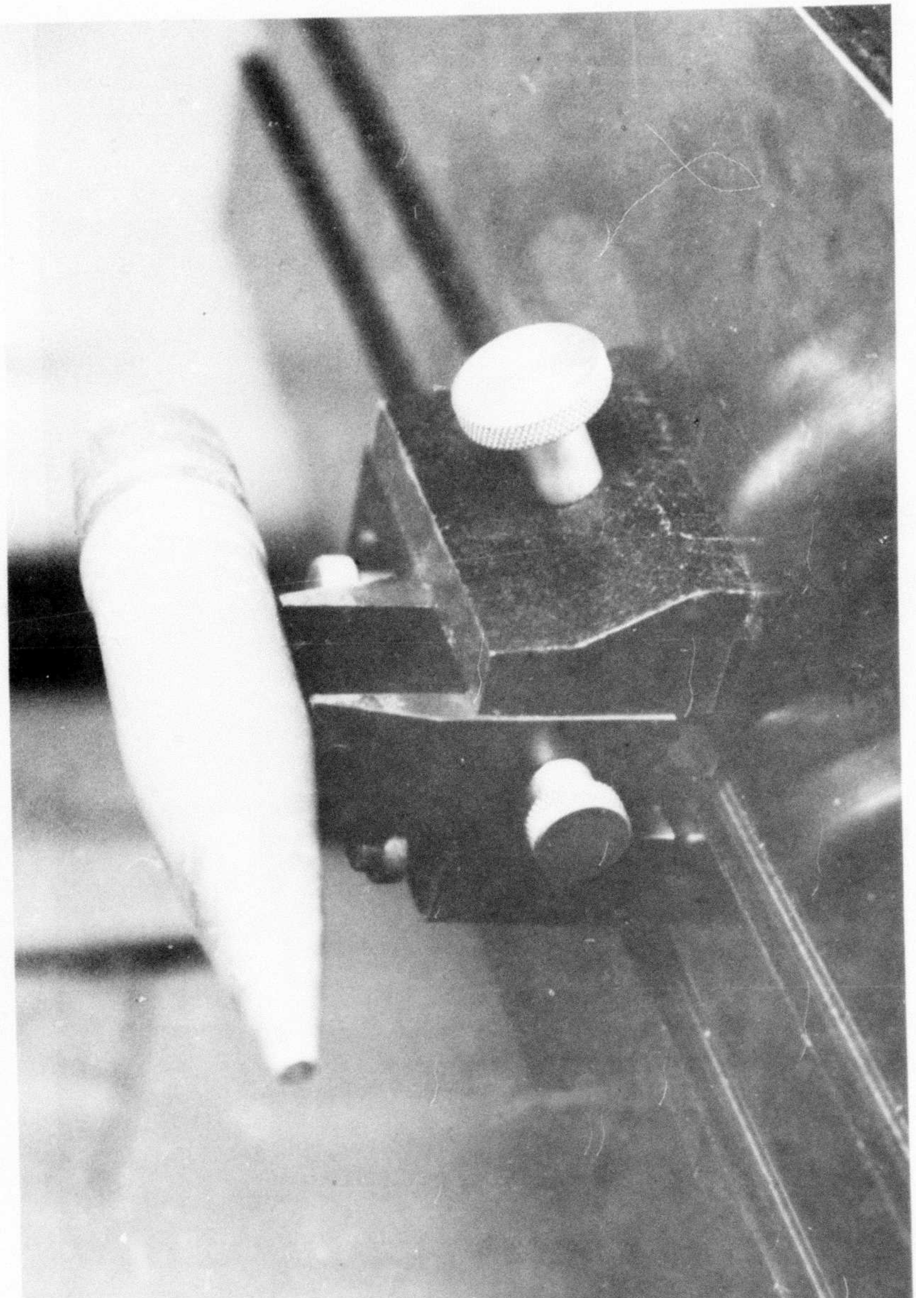


Figure 26. Screw Machine Products Stage for Optical Comparator

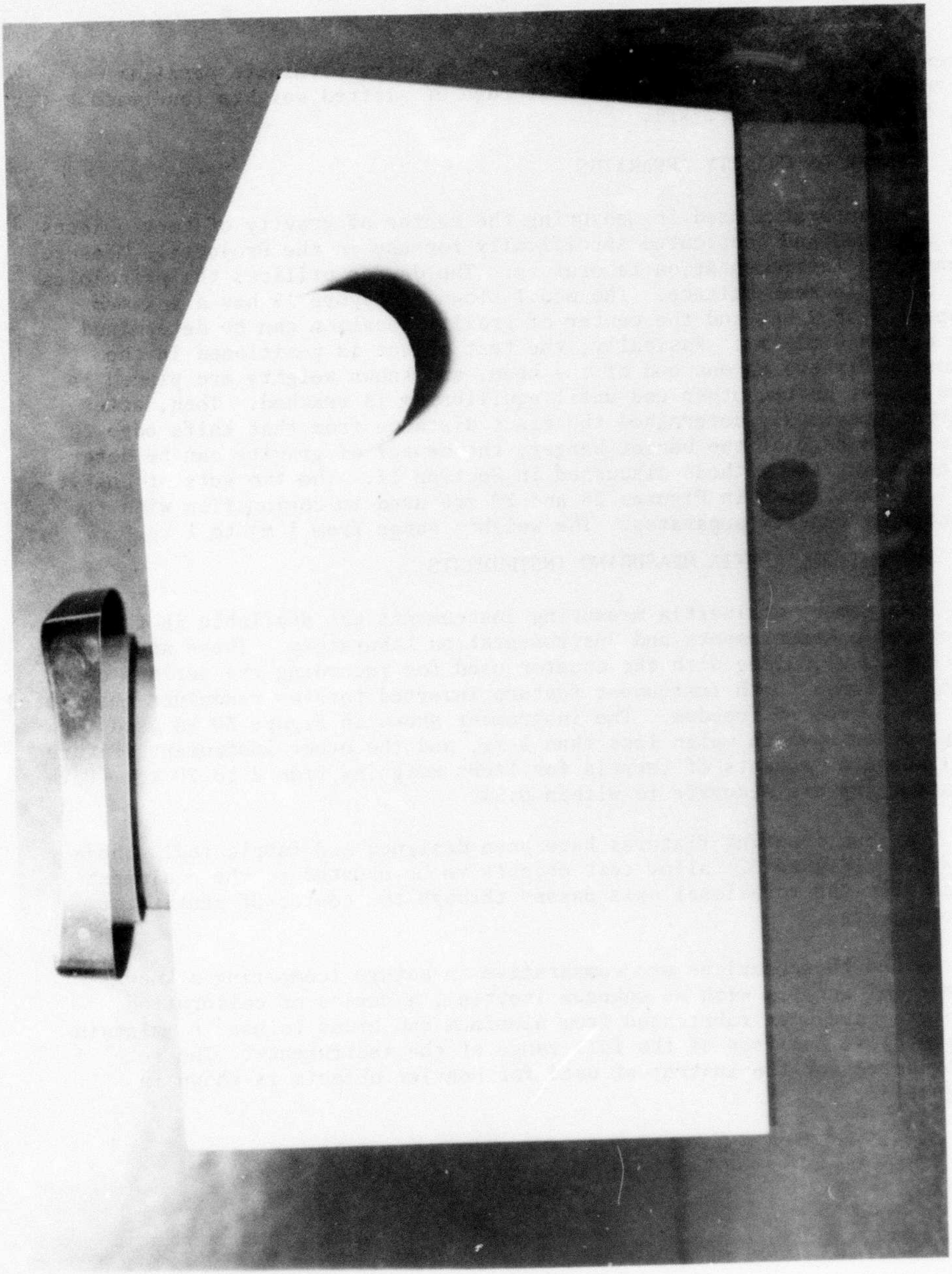


Figure 27. Left Side View of 1 kg Balance

accurate readout from front or rear. This helps eliminate parallax errors. A built-on rack permits storage of slotted weights (one each 1, 5, and 10 kg and 2 kg).

## 3. CENTER OF GRAVITY APPARATUS

The apparatus used in measuring the center of gravity of test objects is designed and fabricated specifically for use in the Projectile Measurements and Instrumentation Laboratory. The device utilizes the principles of a simple beam balance. The model shown in Figure 15 has a maximum capacity of 2 kg, and the center of gravity location can be determined to within 0.025 mm. Basically, the test object is positioned in the housing fixture on one end of the beam, and known weights are placed in the basket on the other end until equilibrium is reached. Then, after having previously determined the exact distance from that knife edge to the knife edge of the basket hanger, the center of gravity can be determined using the methods discussed in Section II. The two sets of analytical weights shown in Figures 28 and 29 are used in conjunction with the center of gravity apparatus. The weights range from 1 mg to 1 kg.

## 4. MOMENT OF INERTIA MEASURING INSTRUMENTS

Two moment of inertia measuring instruments are available in the Projectile Measurements and Instrumentation Laboratory. These are shown in Figure 21, along with the counter used for recording the periods of oscillations. Both instrument feature inverted torsion pendulums restricted to one degree of freedom. The instrument shown in Figure 30 is used for test objects which weigh less than 2 kg, and the other instrument (Figure 31) measures moments of inertia for items weighing from 2 to 20 kg. Both instruments are accurate to within 0.5%.

Various mounting fixtures have been designed and fabricated. These fixtures (Figure 32) allow test objects to be mounted to the equipment such that the rotational axis passes through the center of gravity of the objects.

Since these devices are comparative in nature (comparing a known moment of inertia with an unknown inertia), a series of calibration objects have been fabricated from aluminum and brass to use in maintaining a close analysis of the full range of the instruments. The set fabricated for the instrument used for heavier objects is shown in Figure 33.

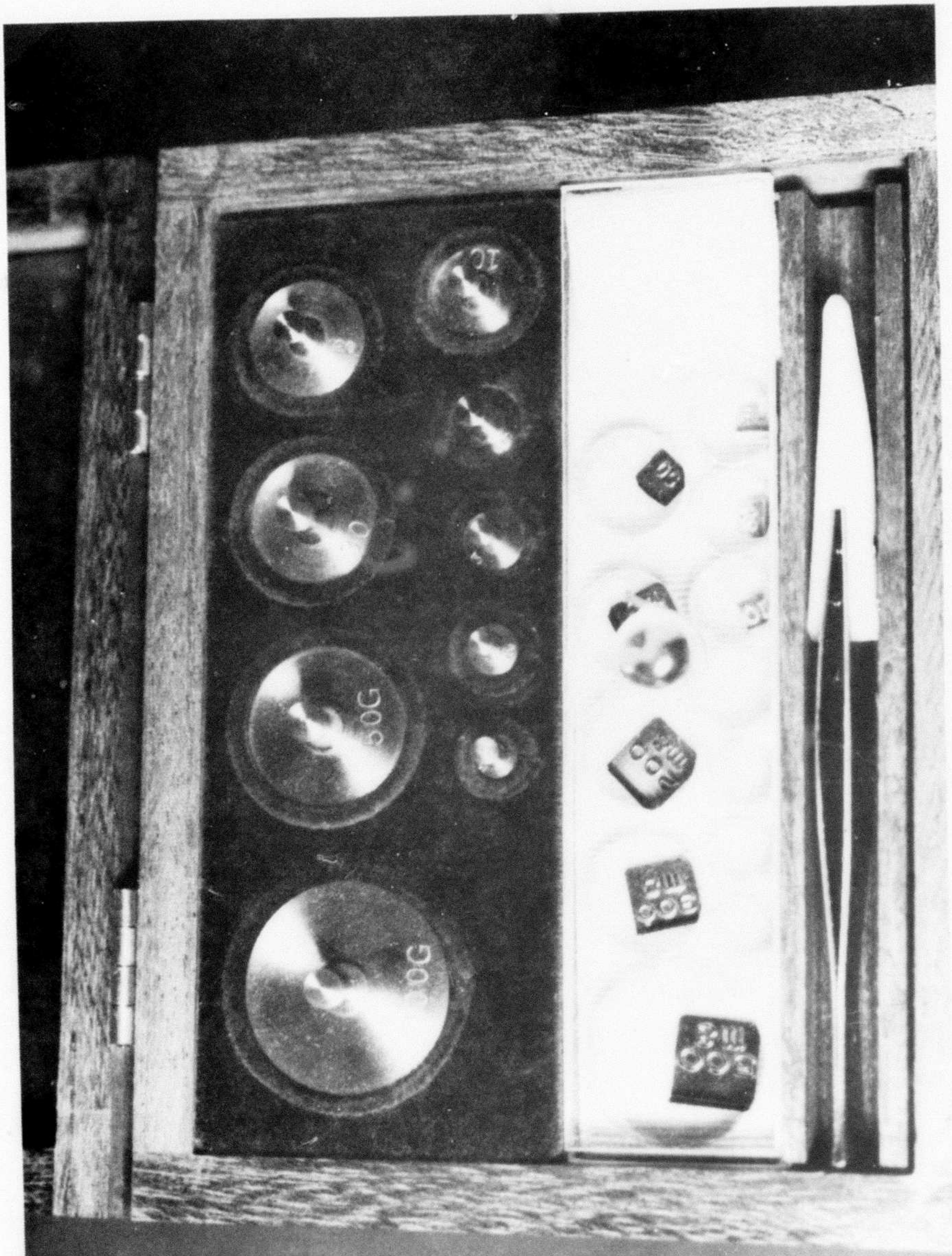


Figure 28. Weight Set: 1 mg to 100 gram

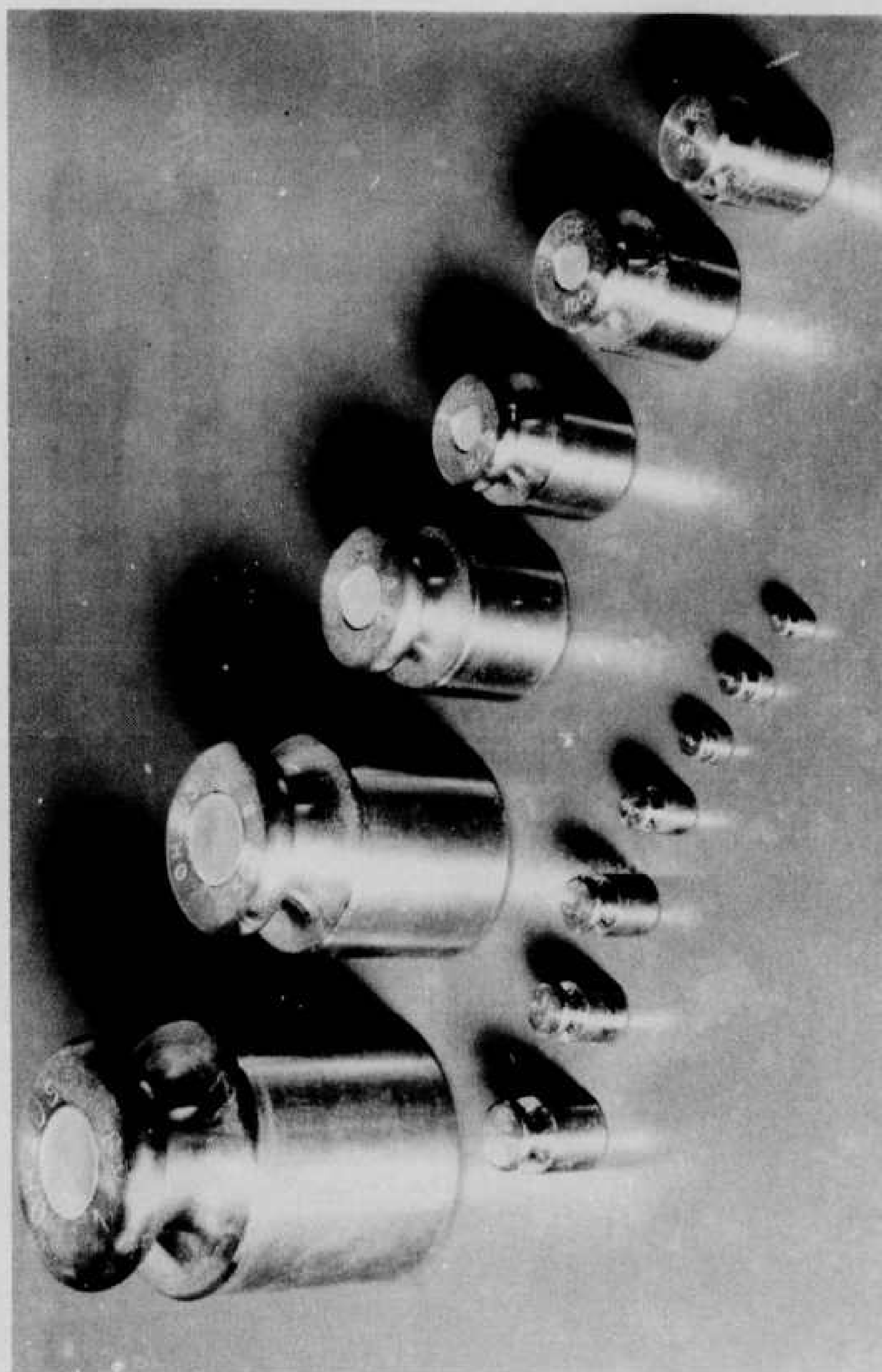


Figure 29. Weight Set: 1 gm to 1 kg

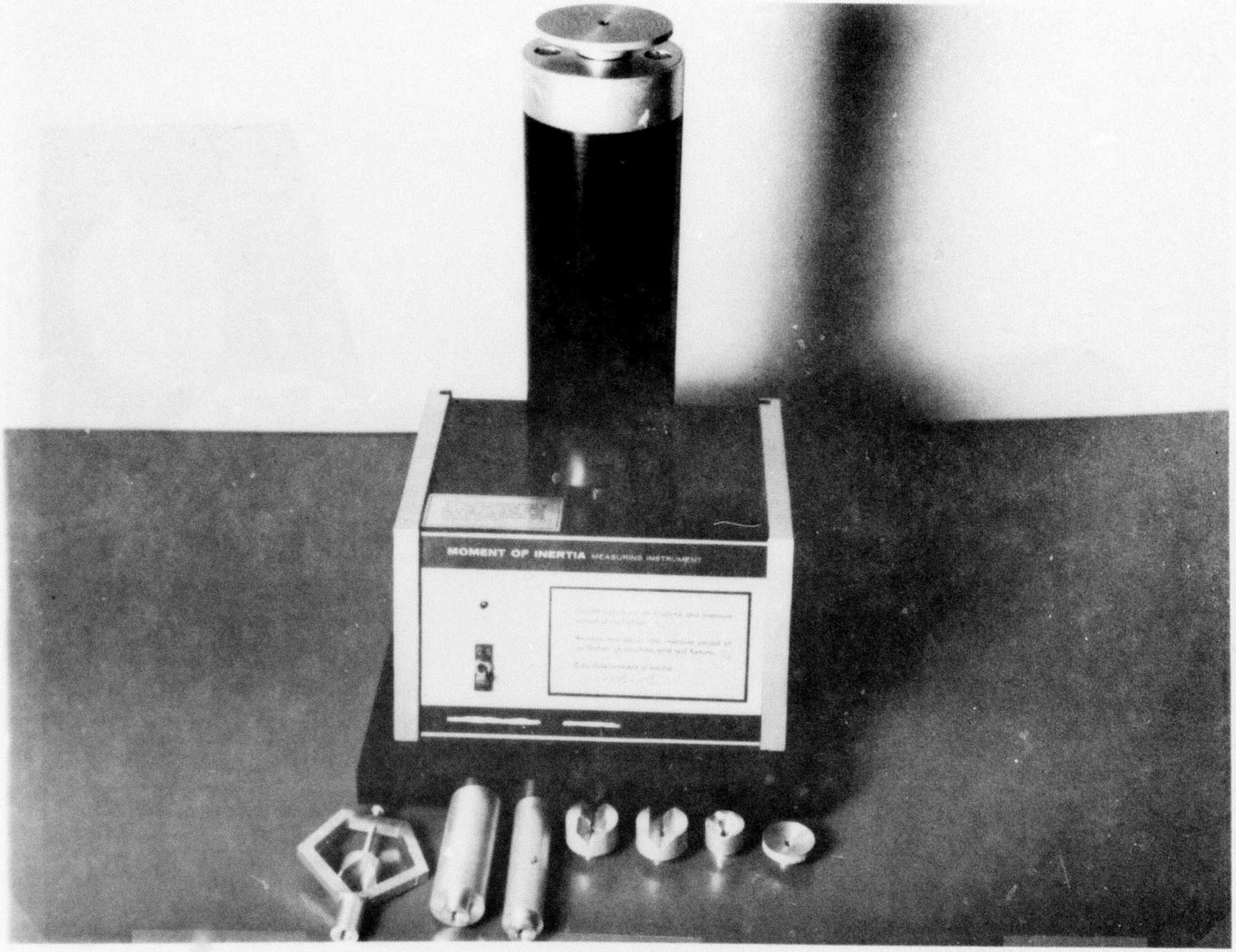



Figure 30. Moment of Inertia Instrument (For objects weighing less than 2 kg)

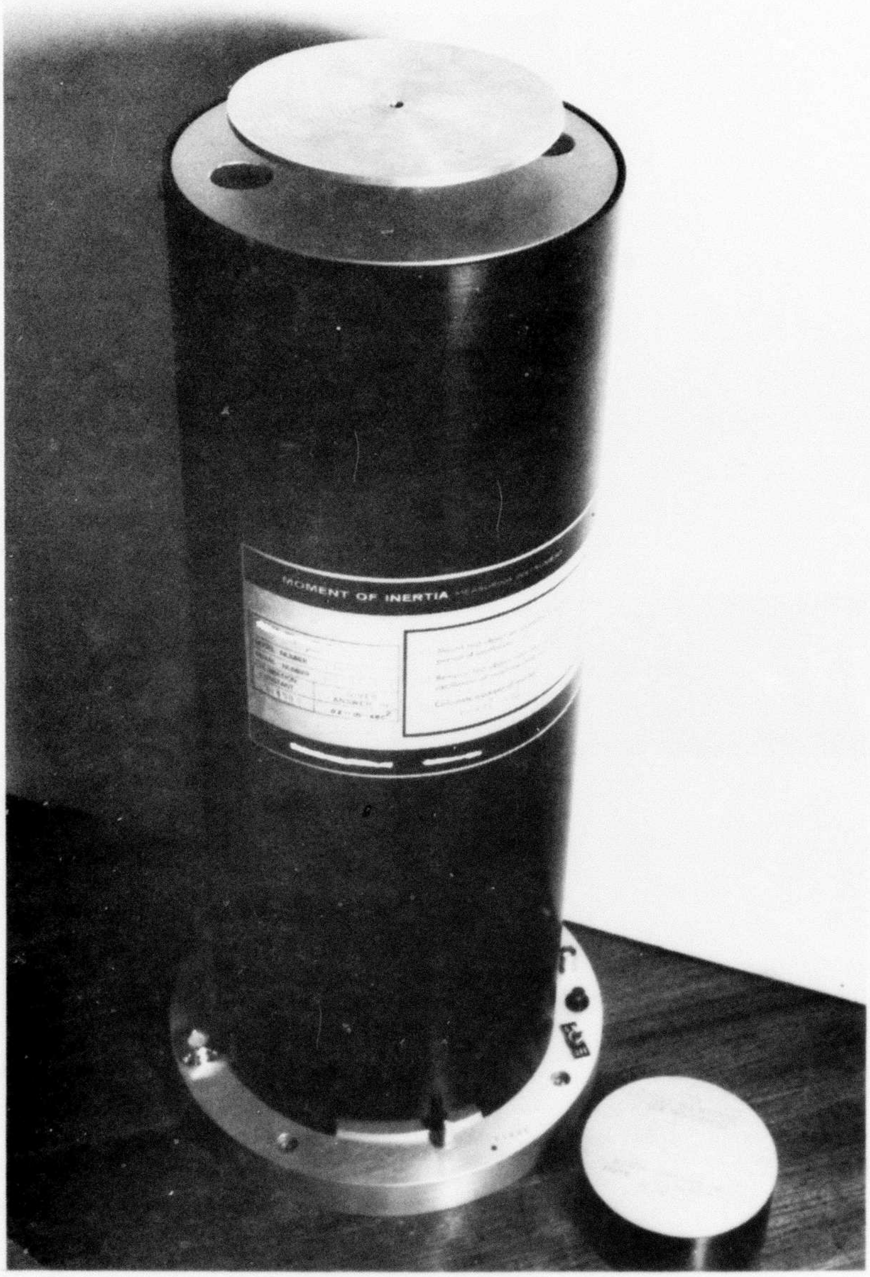



Figure 31. Moment of Inertia Instrument
(For objects weighing from 2 kg to 20 kg)

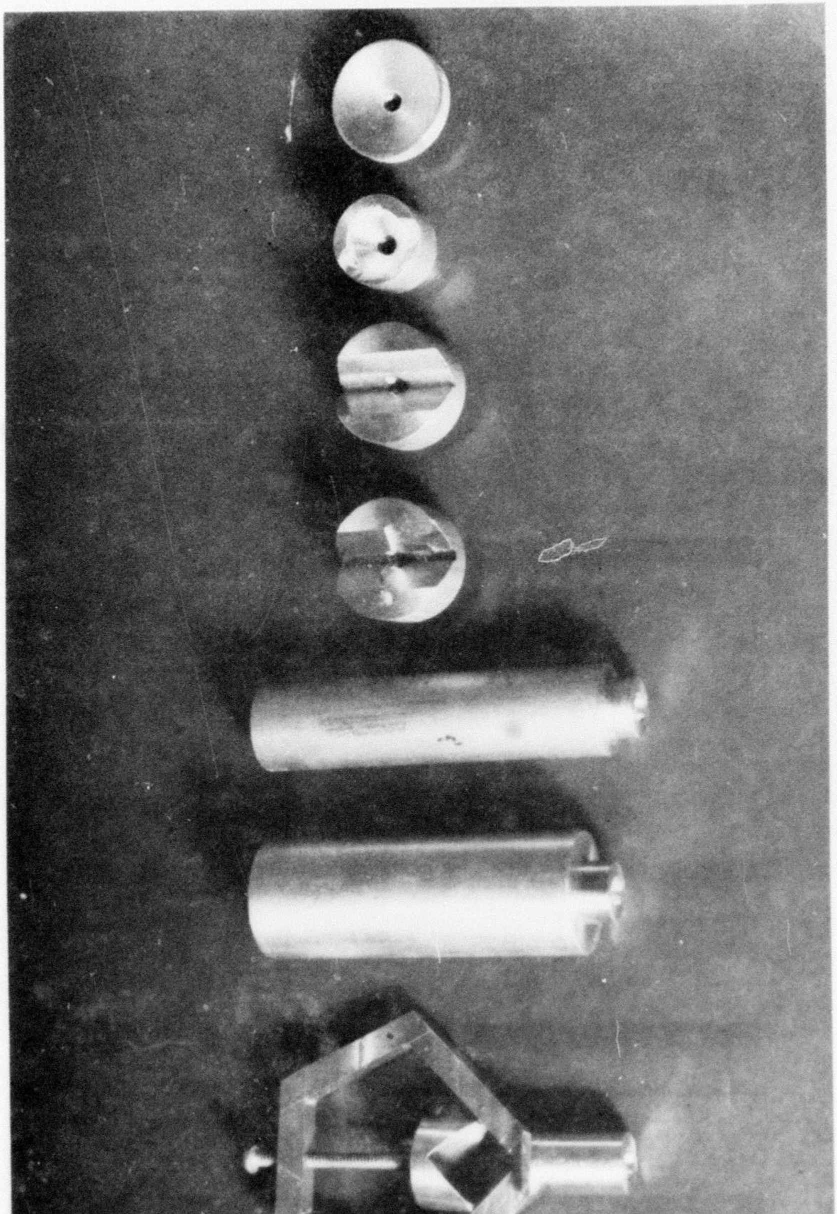
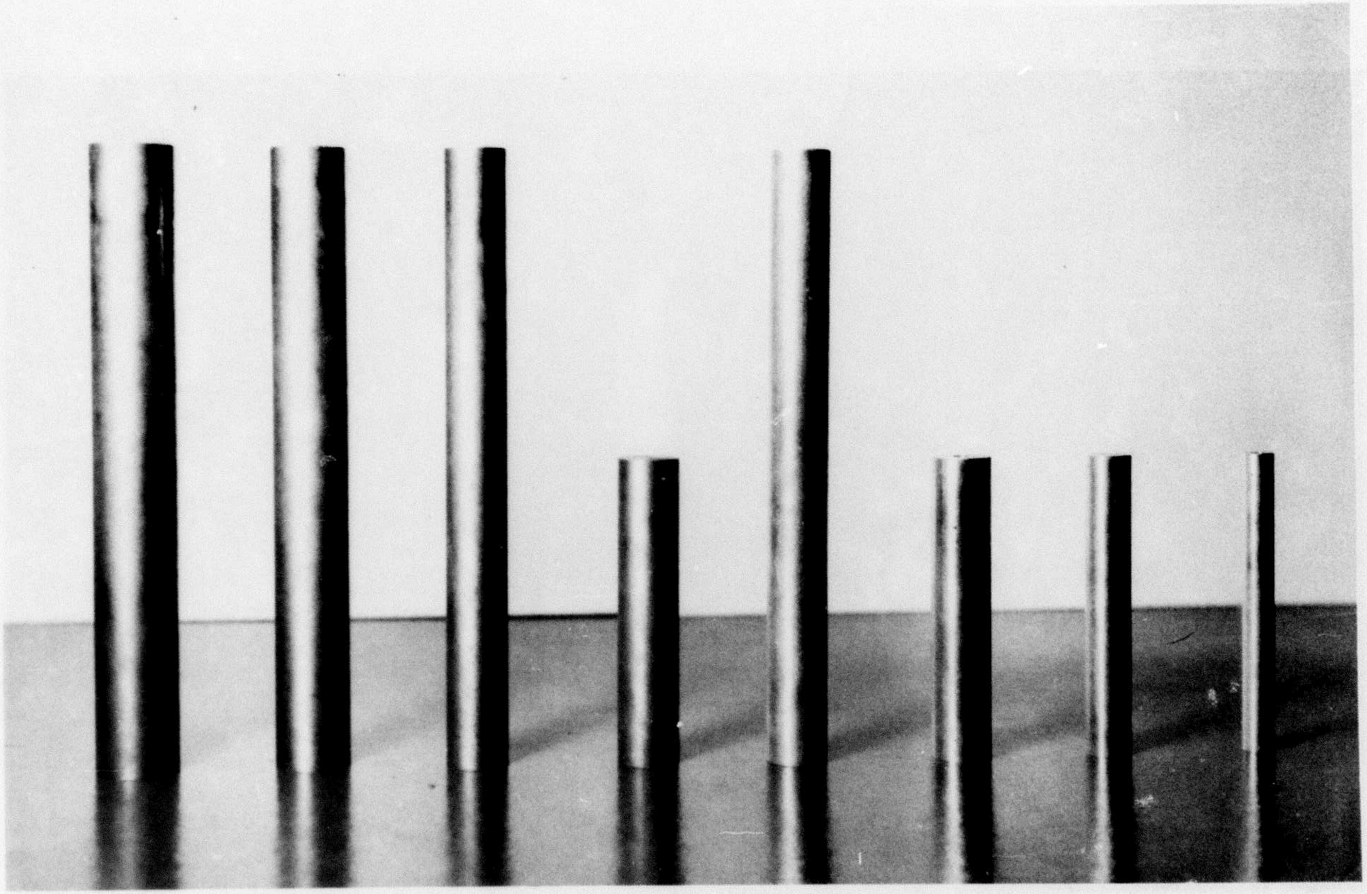


Figure 32. Test Object Mounting Fixtures



Fiugre 33. Moment of Inertia Calibration Cylinders

## SECTION IV

## MEASUREMENT ANALYSIS: PHILOSOPHY AND FUTURE PLANS

The foregoing sections discussed the significance of mass property measurements and how they are accomplished. This section presents an analysis of needed improvements in the measuring process and/or measuring instruments.

The simplest method of measurement is to identify the unit characteristics in an object and then merely to count the recurrences. Almost equally simple are the cases where the unit can be directly compared, as in measurement of length or weight. More advanced measurements, such as those required in aerodynamics, require the construction of chains of relationships. Even when systems are demonstrably weak, the difficulties of retraining and reeducating the users of data, to say nothing of the problem of development of a better system, frequently hinder improvements.

Without standardized data collection procedures, all relevant information about the time, place, persons, etc, in addition to the data itself, would have to be reported. This in turn leads to a consideration of the accuracy of the measurement.

Accuracy is itself a measurement, that is, the measurement of the degree to which a given measurement may deviate from the truth. No procedure can be called a measurement unless it includes methods of estimating accuracy. In statistical literature, accuracy is sometimes defined in terms of a confidence interval. In so far as this computed interval has any meaning, it relates that a certain range of numbers constructed out of observations has a specific probability of including the true measurement. Each set of observations is the basis for forming a net to "catch" the truth, and the confidence interval indicates the probability of a successful catch. It is usually difficult to determine how the information supposedly contained in a confidence interval can be used; i.e., what difference would it make if the confidence interval were twice as large, or half as large. In considering this question, most statisticians seem to urge the decision maker to set his own size of confidence interval. Since most decision makers do not understand the purpose of the interval in the first place, the interval is set arbitrarily, i.e., pointlessly.

Control of the measuring process is the long-run aspect of accuracy and provides the guarantee that measurements can be used in a wide variety of contexts. In other words, a control system for measurement provides optimal information about the legitimate use of measurements under varying circumstances. The economics of control are extremely difficult to develop. It is certainly not economical to check measurements at every feasible instant, nor is it economical to use measurements without any check. Determining the proper amount of control and its structure is in general an unsolved problem. Control is, in effect, the test of a good set of standard procedures. If adjustments can satisfactorily be made to a set of procedures in accordance with the control criteria, then the standards for the procedures have been sufficiently specified. If not, then either the adjustments must be changed or additional specifications must be added to the standards.

In any measurement laboratory, a definite system must be established to ensure the accuracy of the instruments. The responsibility for calibration and control of these instruments must be definitely assigned and enforced. That is to say, all instruments required for mass property measurements should be kept in a controlled environment under the auspices or care of a technician who is qualified to calibrate, adjust, and make minor repairs.

The selection criteria for a special measurement instrument have been discussed in Section III. These factors must be considered for design and development of new equipment. In the planning stage are various instruments to be fabricated to give the complete inclusive range for determining the center of gravity of objects that weigh up to 45 pounds. These instruments are expected to make further use of electrical means of recording data points.

APPENDIX A

# CONVERSION FACTORS

## 1. SYSTEM OF UNITS

Four basic systems of units are used to express moment of inertia. Two of these are gravitational systems which consider force or weight as a basic quantity, and the other two are absolute systems, which consider mass as a basic quantity. Engineers generally use gravitational systems because weight is easy to measure, but physicists use absolute systems. Thus, when an engineer speaks of 5 pounds, he is referring to the weight of an object, but a physicist would talk about 5 pounds mass. Although the units of measurement have different meanings in the two systems, there is generally a minimum amount of confusion when discussing moment of inertia, since the combination of units used is not the same. For example, lb-in$^2$ can only be moment of inertia in the British absolute system and lb-in-sec$^2$ can only be moment of inertia in the British gravitational system, etc. In fact, some engineers use several systems of units without realizing that one system has pound mass and another has pound weight as the fundamental quantity.

SYSTEMS OF UNITS

| TYPE | GRAVITATIONAL | | ABSOLUTE | |
|---|---|---|---|---|
| | Weight is a fundamental quantity, mass = weight/gravity | | Mass is a fundamental quantity | |
| | British | MKS | British | CGS |
| FORCE (wt) | POUND | kg | POUNDAL | DYNE |
| MASS | SLUG | kg-m-sec$^2$ | POUND | GRAM |

## 2. CONVERSION CHART

a. To convert to oz-in-sec$^2$, multiply by the appropriate conversion factor from the following chart:

b. To convert from oz-in-sec$^2$ to a new set of units, divide by the appropriate conversion factor.

| UNITS | CONVERSION FACTOR | SYSTEM USED |
|---|---|---|
| lb-in$^2$<br>lb-ft$^2$<br>oz-in$^2$ | $4.145 \times 10^{-2}$<br>5.967<br>$2.59 \times 10^{-3}$ | British Absolute |
| gm-cm$^2$<br>kg-cm$^2$<br>kg-m$^2$ | $1.415 \times 10^{-5}$<br>$1.415 \times 10^{-2}$<br>141.5 | CGS |
| oz-in-sec$^2$<br>lb-in-sec$^2$<br>lb-ft-sec$^2$<br>slug-ft$^2$ | 1.00<br>16.00<br>192.00<br>192.00 | British Gravitational |
| kg-m-sec$^2$<br>gm-m-sec$^2$<br>gm-cm-sec$^2$ | 1385<br>1.385<br>$1.385 \times 10^{-2}$ | MKS |

EXAMPLE I:

A value of 10 oz-in-sec$^2$ is given; convert to lb-in$^2$:

10 oz-in-sec$^2$/$4.145 \times 10^{-2}$ = 241.3 lb-in$^2$

EXAMPLE II:

A value of 0.01 slug-ft$^2$ is given; convert to oz-in-sec$^2$:

0.01 slug-ft$^2$ x 192 = 1.92 oz-in-sec$^2$

EXAMPLE III:

A value of 3 lb-in-sec$^2$ is given; convert to lb-in$^2$:

step (a) 3 lb-in-sec$^2$ x 16 = 48.00 oz-in-sec$^2$

step (b) 48.00 oz-in-sec$^2$/$4.145 \times 10^{-2}$ = 1158 lb-in$^2$

## Bibliography


Alexander, J.; Pomeranz, K.; Prince, J.; Sacher, D.: Physics for Engineering Technology. John Wiley & Sons, Inc., 1966.

Bennet, C. E.: Physics Problems. Barnes & Noble, Inc., 1958.

Bevington, P. R.: Data Reduction and Error Analysis for the Physical Sciences. McGraw-Hill, 1969.

Daniel, D. C.; Winchenbach, G. L.; Browell, E.V.: The USAF Aeroballistic Research Facility. USAF Armament Laboratory, September 1973.

DeGroot, A.: Practical Guide to Mechanics. Theodore Andel & Co., 1964.

Dickinson, E. R.: Physical Measurements of Projectiles, Technical Note No 874. Ballistic Research Laboratories, February 1954.

Domholdt, L. C. and Tune, G. L.: Engineering Experimentation. McGraw-Hill, 1966.

Housner, G. W. and Hudson, D. E.: Applied Mechanics Statics, 2nd Ed, 1961, D. VanNostrand.

Margenan, H. and Murphy, G. M.: The Mathematics of Physics and Chemistry. VanNostrand, 1943.

Moffat, R. J.: Uncertainty Analysis. Stanford University, August 1970.

Morin, R. M.: Mechanical Plus Optical. Automatic Machining, February 1969.

Palmer, C. H.: Optics: Experiments and Demonstrations. John Hopkins Press, 1962.

Parker, H.: Simplified Mechanics and Strength of Materials, 2nd Ed, 1961. John Wiley & Sons, Inc.

Perry, R. H.: Engineering Manual, 2nd Ed, 1967. McGraw-Hill.

Rusk, R. D.: Introduction to College Physics. Appleton-Century-Crofts, Inc., 1954.

Sholil, L. L. and Smith, L. E.: Modern College Physics. Wm. C. Brown Co., Inc., 1960.

Sonders, M.: The Engineers Companion. John Wiley & Sons, Inc., 1967.

Stout, M. B.: Basic Electrical Measurements, 2nd Ed, 1960. Prentice-Hall.

Sweeney, R.J.: Measurement Techniques in Mechanical Engineering. John Wiley & Sons, Inc., 1953.

Wilson, E. B.: An Introduction to Scientific Research. Mc-Graw-Hill, 1952.

Winchenbach, G.L. and Watt, R.M.: Apparatus for Determining the Center of Gravity and Moments of Inertia of an Aeroballistic Model, Test Report AEDC-TM-68-19. Arnold Engineering Development Center, June 1965.

Wood, B. D.: Data Analysis: Propagation of Errors into Final Results. Arizona State University, March 1974.

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